Well-balanced shock-capturing hybrid finite volume-finite difference schemes for Boussinesq-type models

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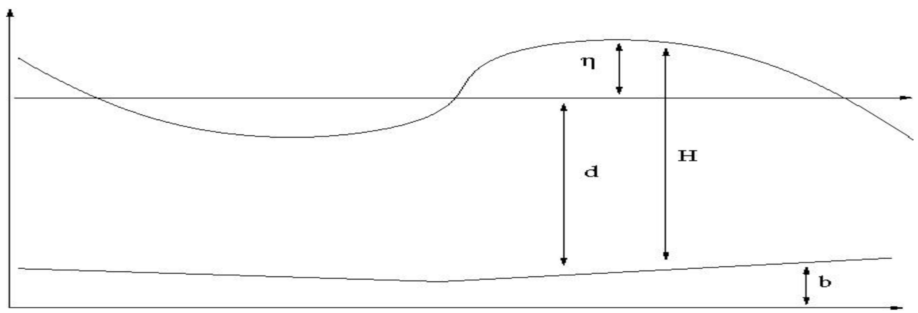
* Most popular (applied): Nonlinear Shallow Water Equations (SWE)
  - Limitation: Not applicable for wave propagation in intermediate/deeper waters (dispersion has an effect on free surface flow)
* Use of popular Boussinesq-type models (but in conservation law from)
  - Nowgu's equations (Nowgu, 1993)
  - Madsen and Sörensen's (MS) equations (Madsen and Sörensen, 1992)

Both have good linear accuracy to $kd \approx 3$ (intermediate water).



η: free surface elevation
b: topography
d: steel water level
$H = \eta + d$: total water depth

Some recent relevant works

* Hybrid finite-volume (FV) finite-difference (FD) schemes to:
  – MS, and Beji and Nadaoka (1996) equations, Roe's Riemann solver, surface gradient method for topography discretization (Zhou et al., 2001), not tested on wet/dry and breaking wave cases (Erduran et al., 2005 and 2007),

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  - Two-layer equation models (Lynnet et al., 2006-2010).

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Vector conservative form (for both models)

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$$\mathbf{U} = \begin{bmatrix} H \\ P^* \end{bmatrix}, \quad \mathbf{F}(\mathbf{U}) = \begin{bmatrix} Hu \\ Hu^2 + \frac{1}{2}gH^2 \end{bmatrix}.$$

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- R_b parametrization of wave breaking characteristics

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- Special treatment wet/dry fronts:
  * Identify dry cells: through an adaptive (grid dependant) tolerance parameter
  * Consistent depth reconstruction: satisfy $\frac{\partial h}{\partial x} = -\frac{\partial b}{\partial x}$ to high-order on wet/dry fronts
  * Satisfy an extended C-property: Redefinition of the bed slope, numerical fluxes are computed assuming temporarily zero velocity at wet/dry faces (Brufau et al., 2004)

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- **Time Integration** (should at least match the order of truncation errors from dispersion terms): Third order Adams-Basforth predictor and fourth-order Adams-Moulton corrector stage (but also tested 3rd and 4th order Runge-Kutta methods).

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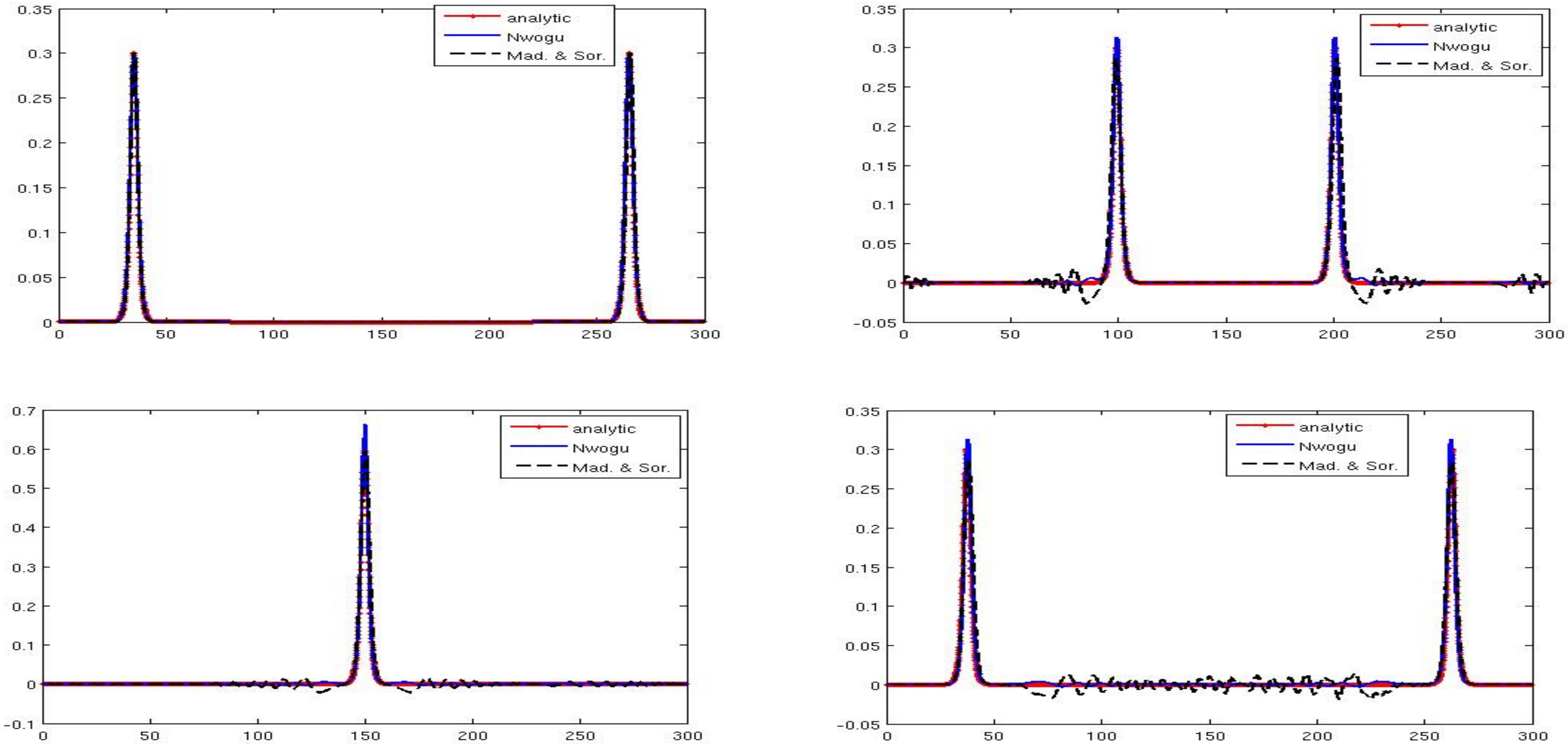
  * Idea: Boussinesq degenerate into NSWE as dispersive terms become negligible (Tonelli and Petti, 2009)
    If $\epsilon = \frac{A}{d} \leq 0.8$ Boussinesq are solved otherwise SWE are solved.

Some Numerical Tests and Results

Head on collision of two solitary waves

Area length $L = [0, 300m]$, Initial hight $A/d = 0.3$, $\Delta x/d = 0.1$, $CFL = 0.2$.

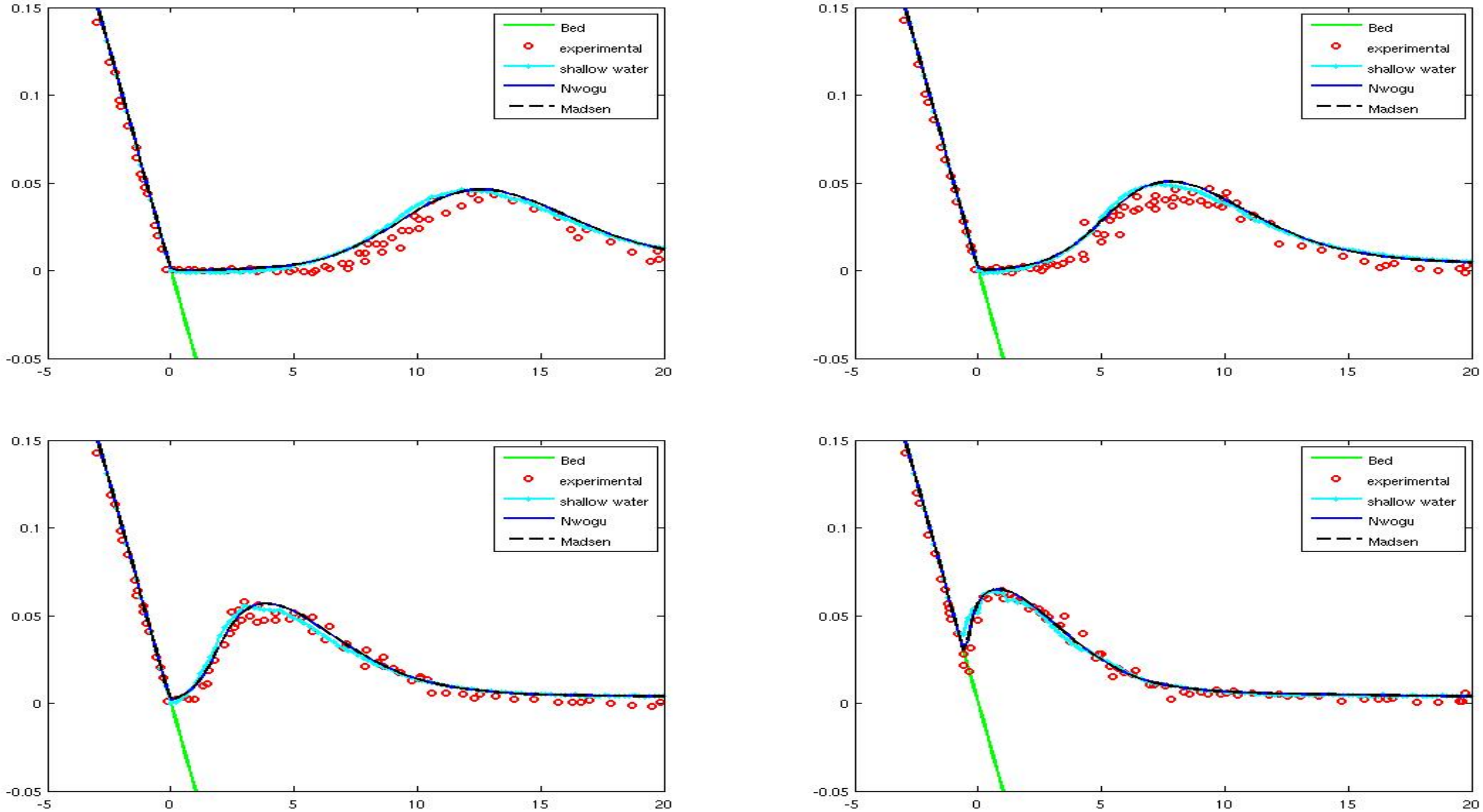


Surface profiles at times $t\sqrt{g/d}$=0, 56.63 ,101.2 and 200.

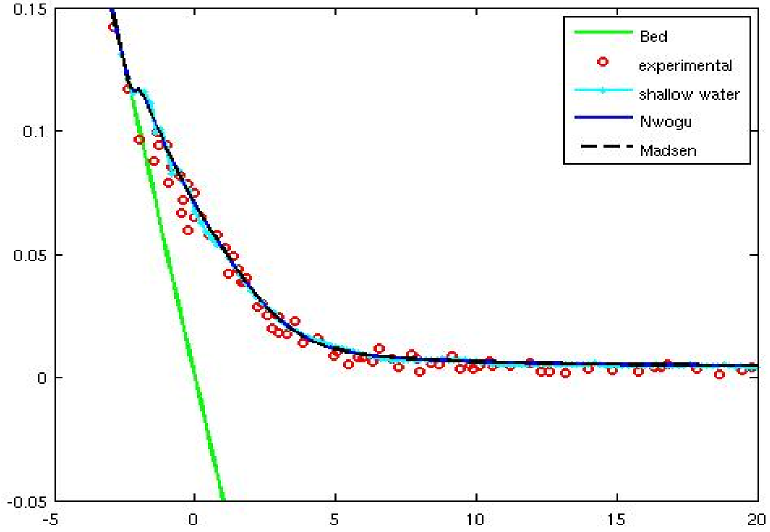
Solitary wave run-up on a plane beach (Synolakis, 1987)

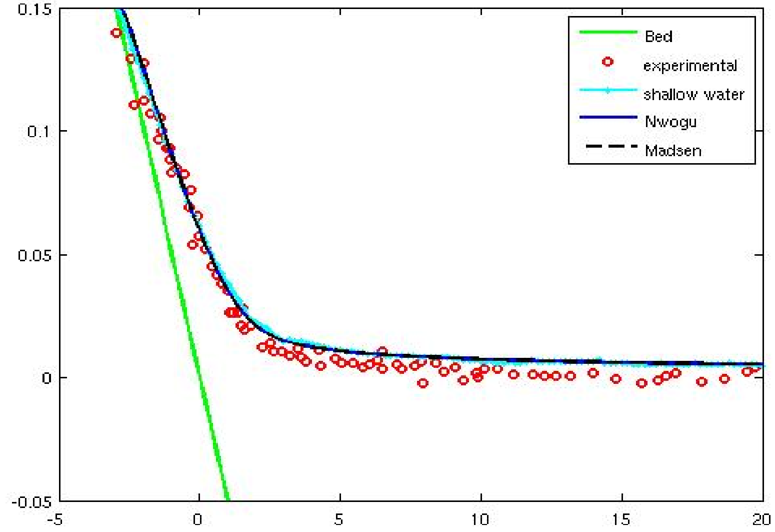
$L = [-10, 100m], \quad \Delta x/d = 0.1, \quad CFL = 0.2, \quad n_m = 0.01,$ slope=1 : 19.85

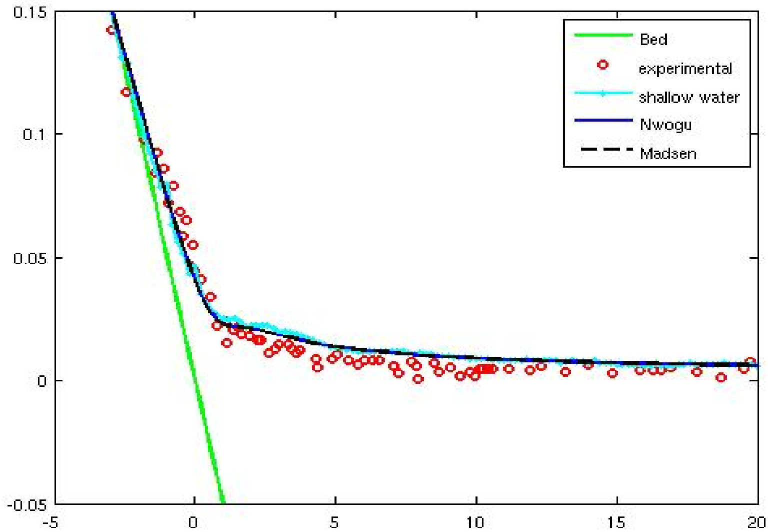
* First case: $A/d = 0.04$ (non-breaking)
* Second case: $A/d = 0.28$ (breaking)

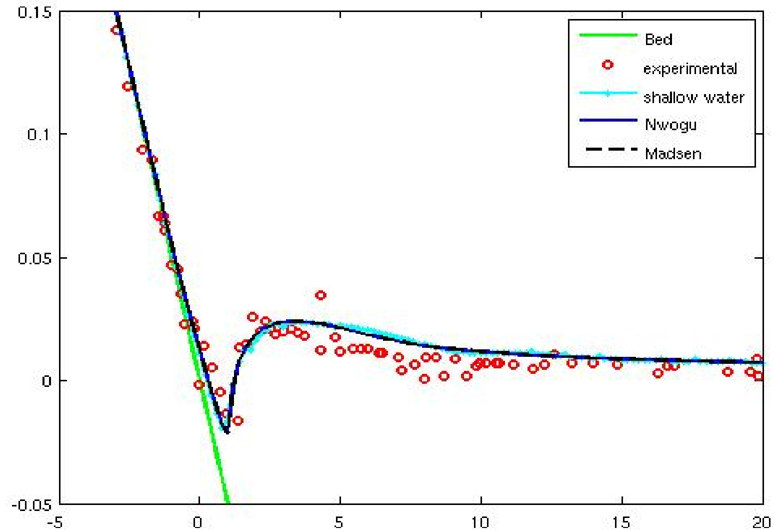


Case A: Surface profiles at times $t\sqrt{g/d} = 20, 26, 32, 38$



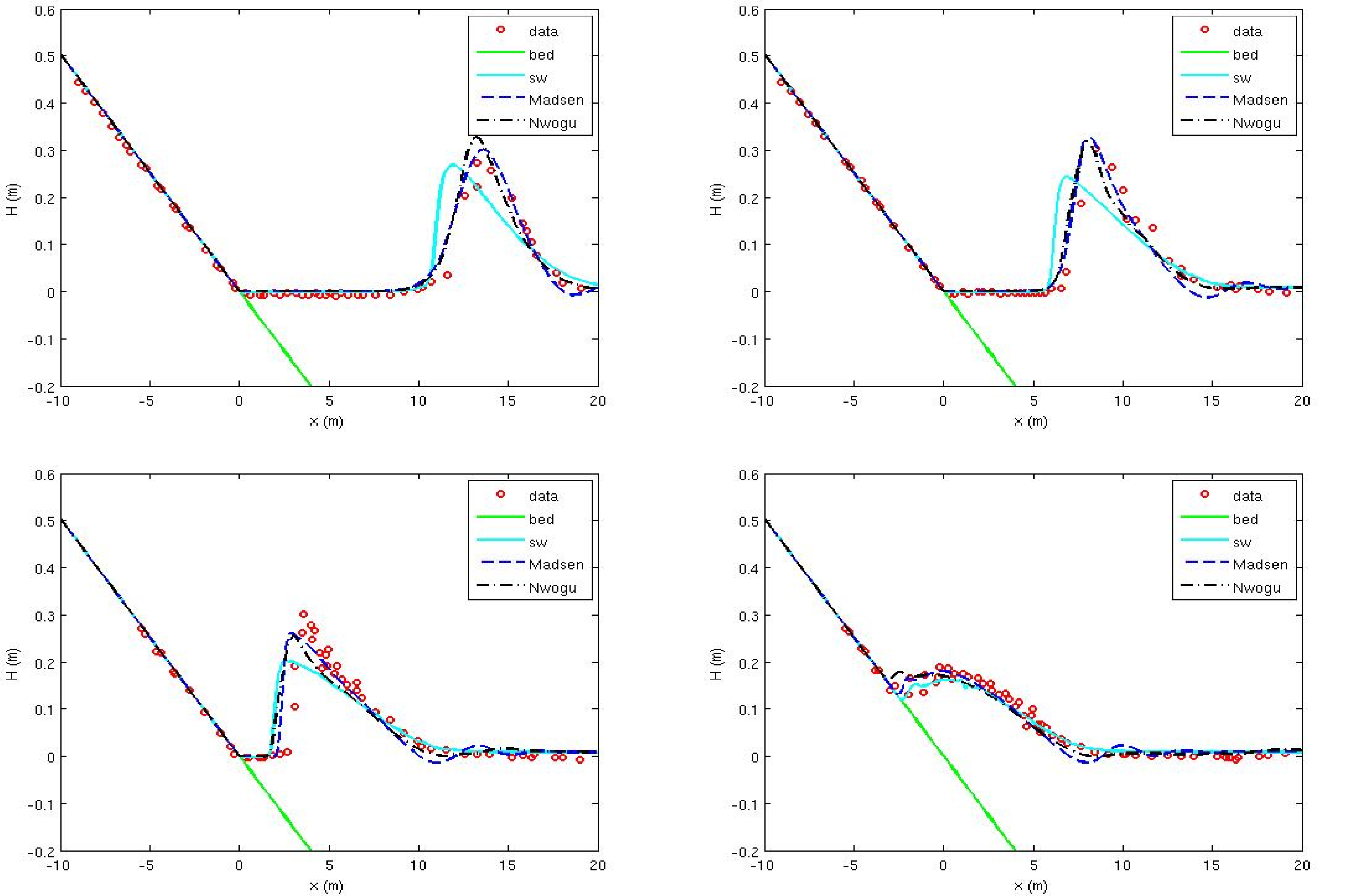






Case A: Surface profiles at times $t\sqrt{g/d} = 44, 50, 56, 62$

Case B: Surface profiles at times $t\sqrt{g/d} = 10, 15, 20, 25$



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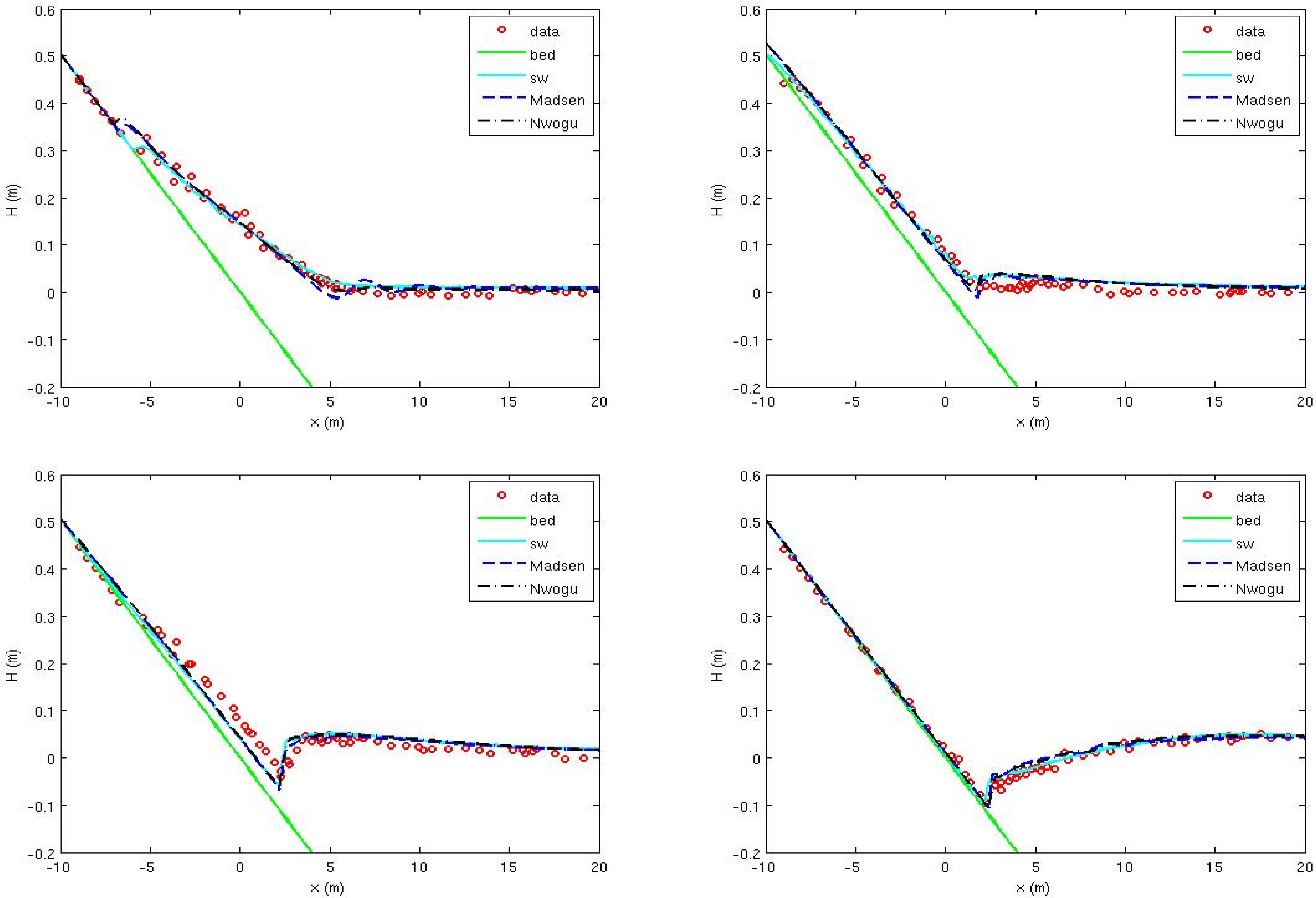
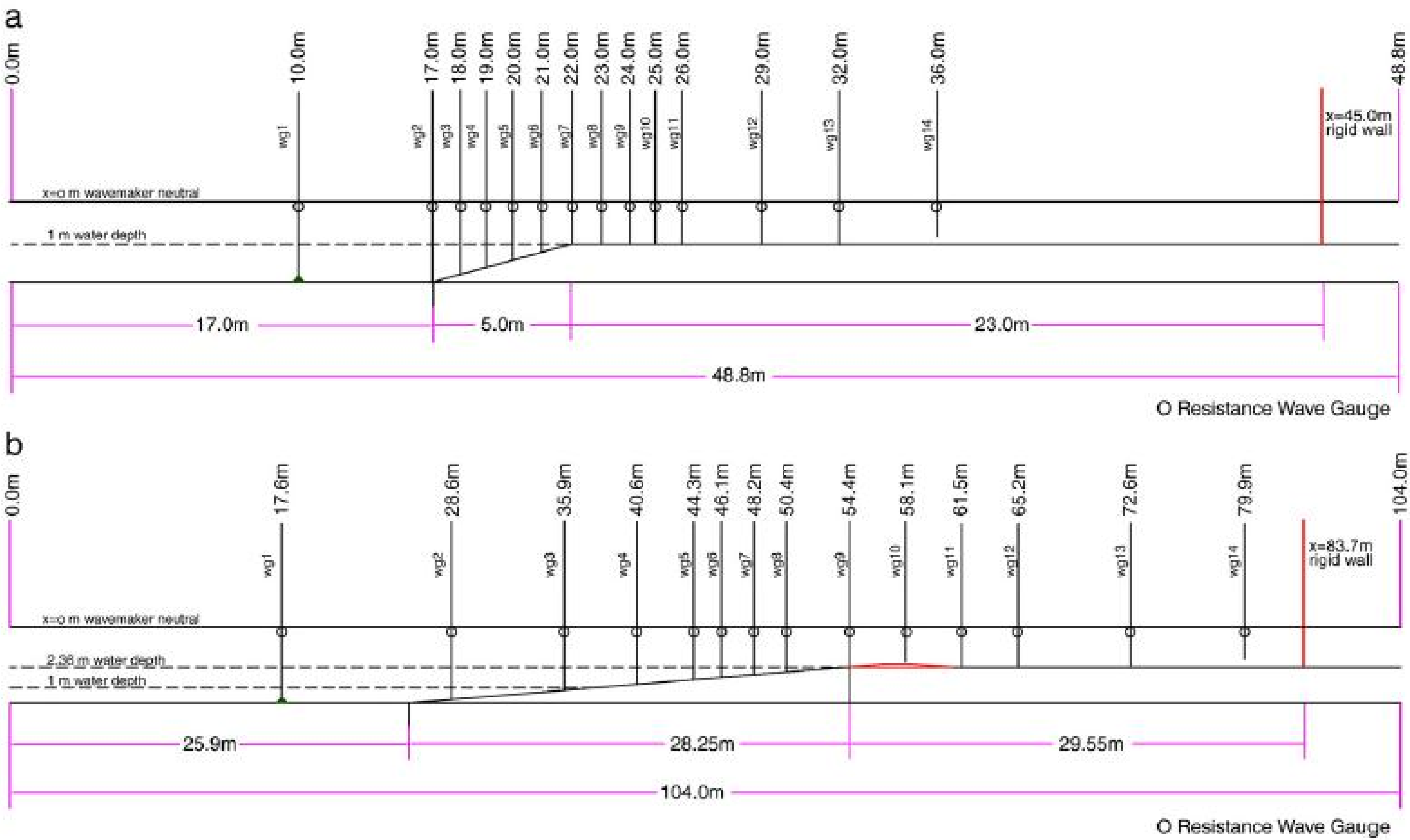


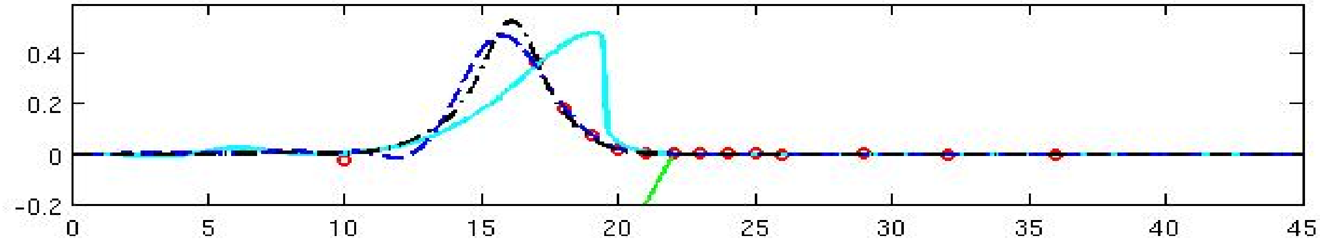
Solitary wave propagation over reefs



Laboratory experiments at the O.H. Hinsdale Wave Research Laboratory of Oregon State University, 2007-2009 (Roeber et al., 2010).

50-cm solitary over a dry reef flat

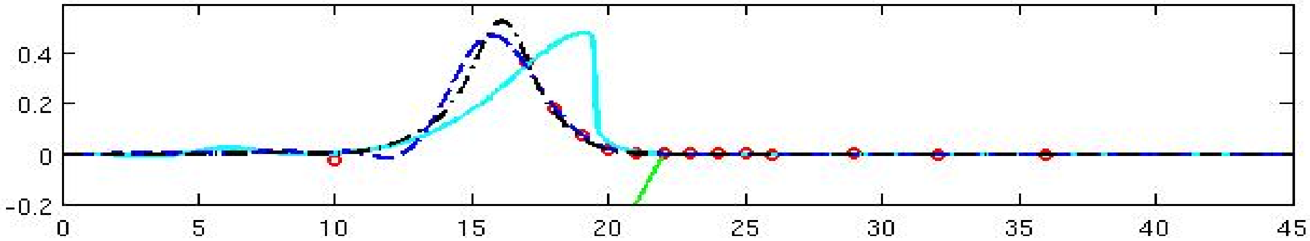
$$L = [0,\ 48.8m],\quad d = 1.0m,\quad \Delta x/d = 0.1m,\quad CFL = 0.4,\quad n_m = 0.012,\quad A/d = 0.5$$



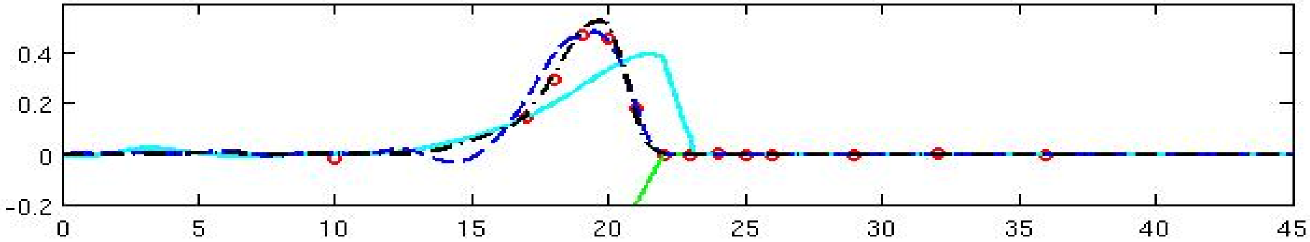
$t = 52.3$ Shoaling

50-cm solitary over a dry reef flat

$$L = [0,\ 48.8m],\quad d = 1.0m,\quad \Delta x/d = 0.1m,\quad CFL = 0.4,\quad n_m = 0.012,\quad A/d = 0.5$$



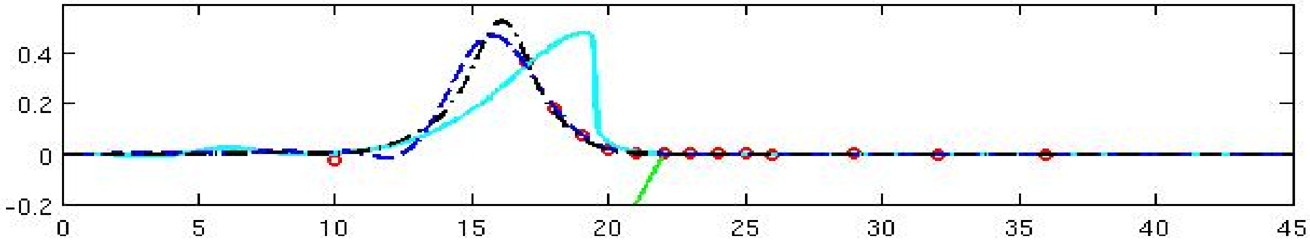
$t = 52.3$ Shoaling



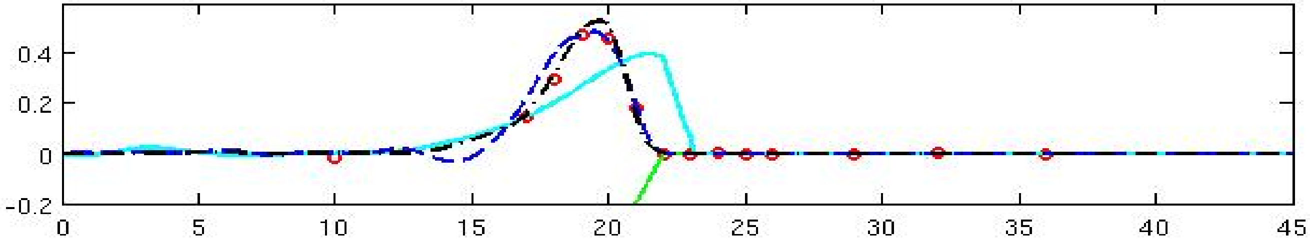
$t = 55.1$ The wave begins to skew

50-cm solitary over a dry reef flat

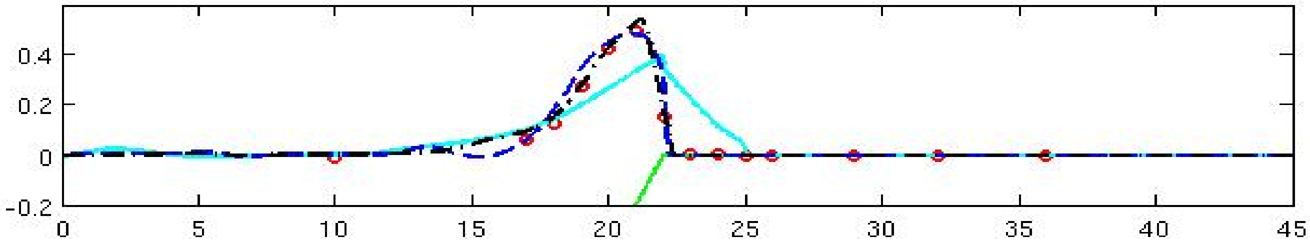
$$L = [0,\ 48.8m],\quad d = 1.0m,\quad \Delta x/d = 0.1m,\quad CFL = 0.4,\quad n_m = 0.012,\quad A/d = 0.5$$



$t = 52.3$ Shoaling



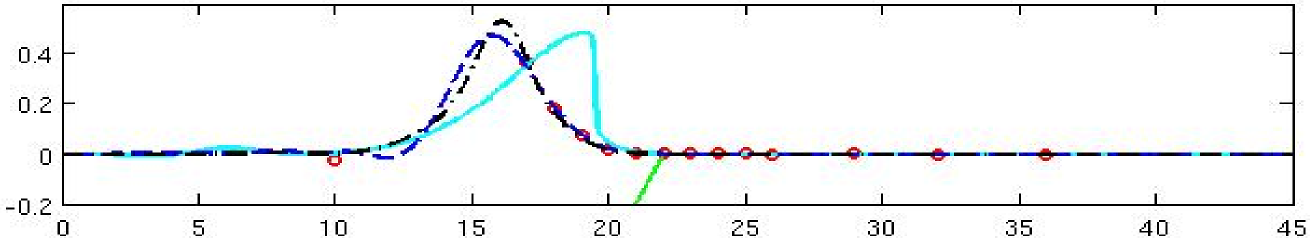
$t = 55.1$ The wave begins to skew



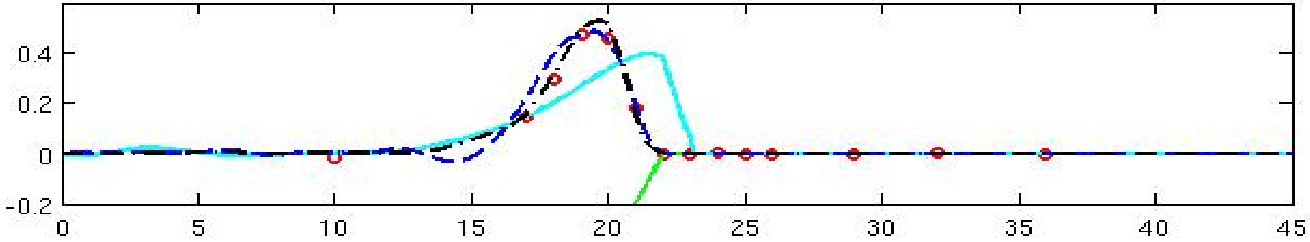
$t = 56.3$ Wave collapse

50-cm solitary over a dry reef flat

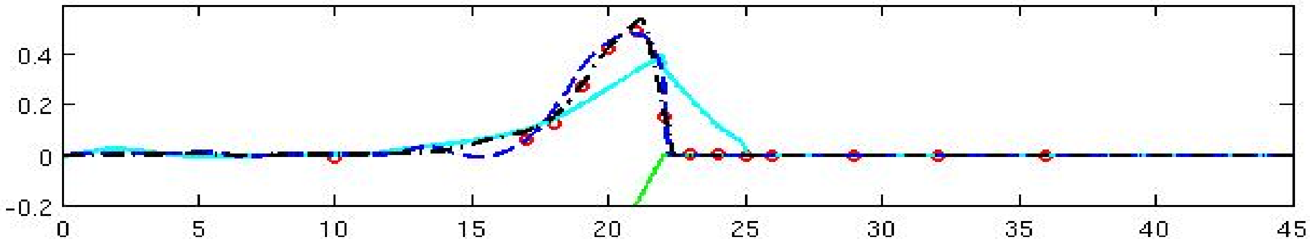
$$L = [0,\ 48.8m],\quad d = 1.0m,\quad \Delta x/d = 0.1m,\quad CFL = 0.4,\quad n_m = 0.012,\quad A/d = 0.5$$



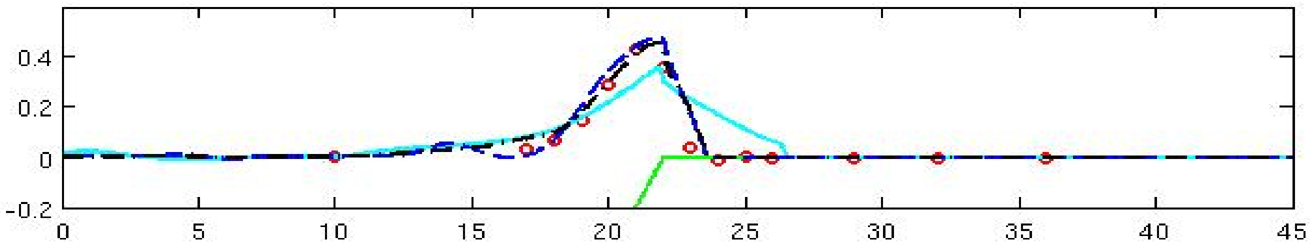
$t = 52.3$ Shoaling



$t = 55.1$ The wave begins to skew



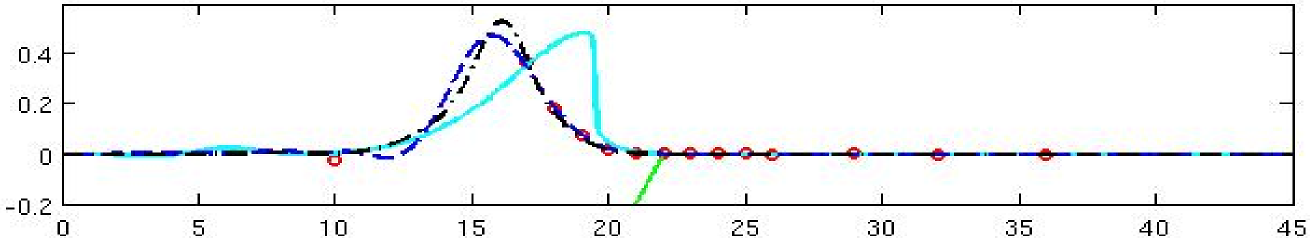
$t = 56.3$ Wave collapse



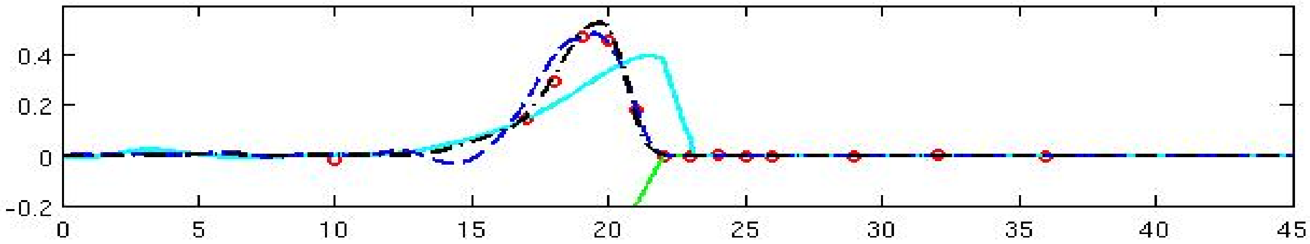
$t = 57.3$ Propagation over the reef

50-cm solitary over a dry reef flat

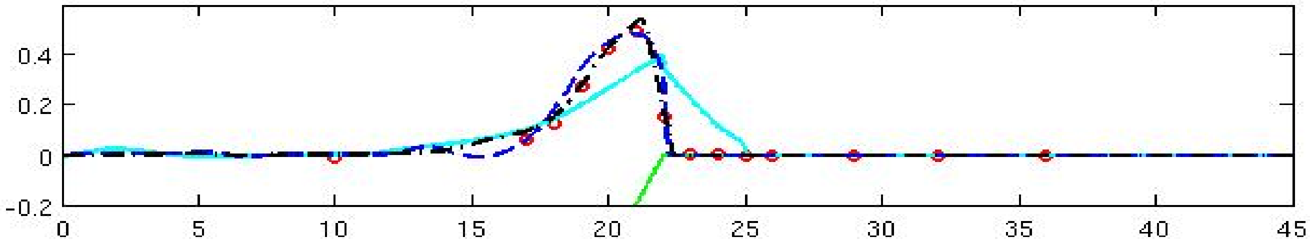
$$L = [0,\ 48.8m],\quad d = 1.0m,\quad \Delta x/d = 0.1m,\quad CFL = 0.4,\quad n_m = 0.012,\quad A/d = 0.5$$



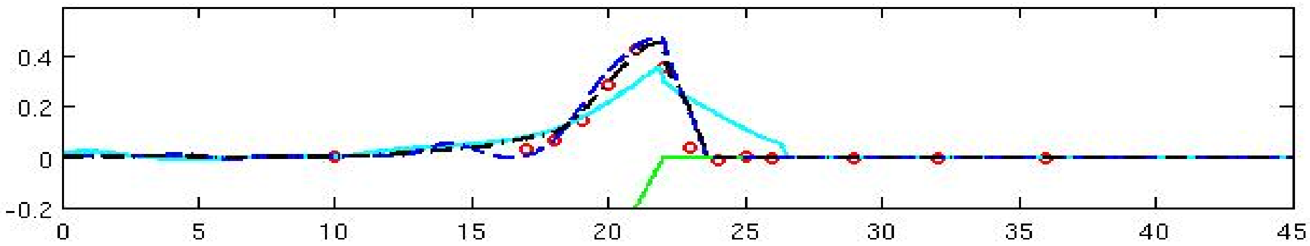
$t = 52.3$ Shoaling



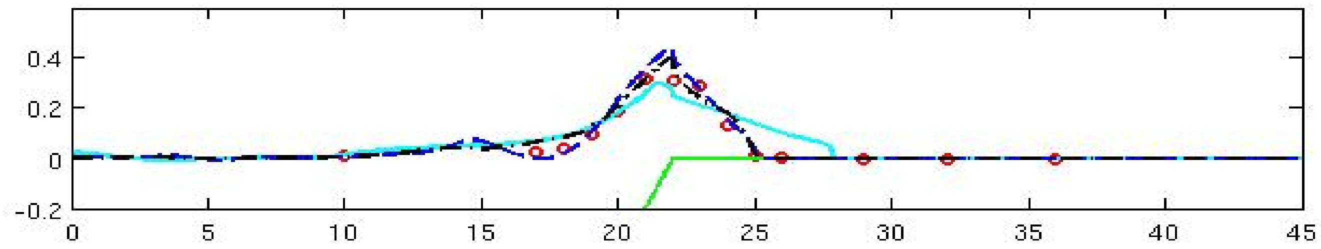
$t = 55.1$ The wave begins to skew



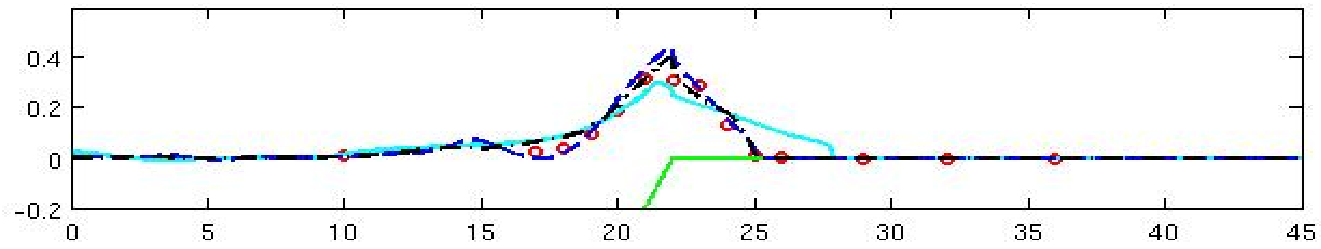
$t = 56.3$ Wave collapse



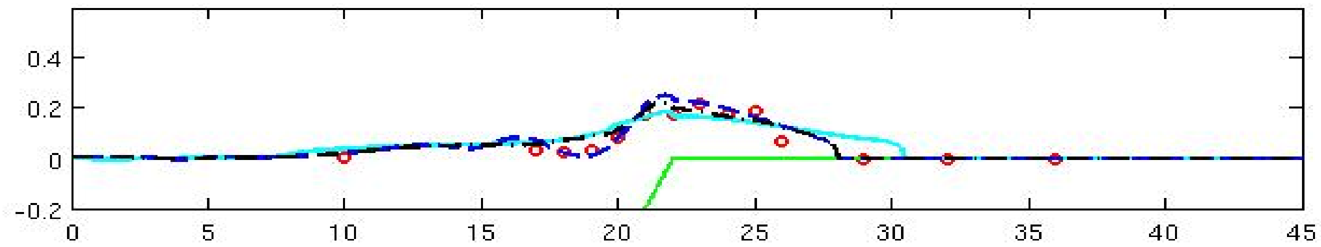
$t = 57.3$ Propagation over the reef



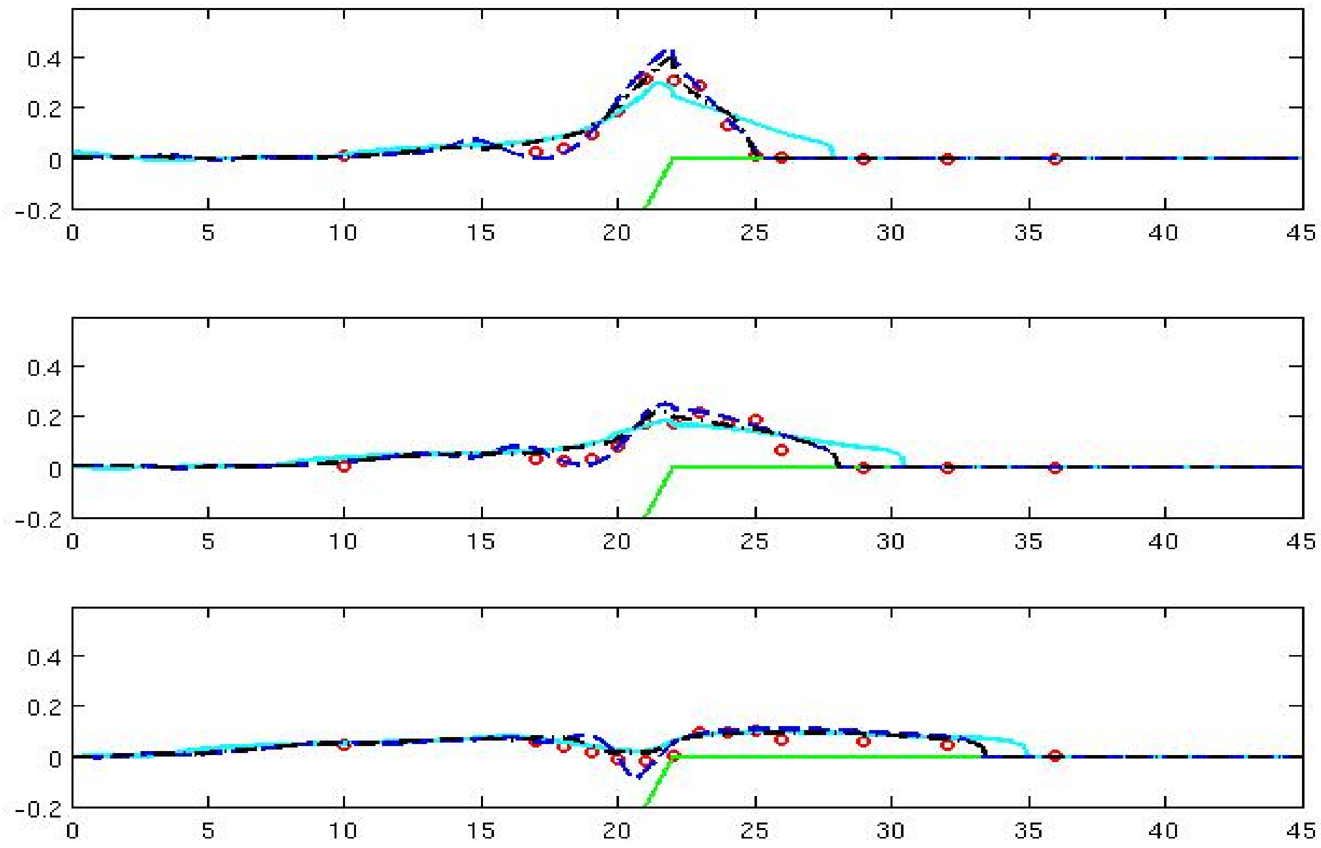
$t = 58.3$


0.4
0.2
0
-0.2
0
5
10
15
20
25
30
35
40
45

$t = 58.3$


0.4
0.2
0
-0.2
0
5
10
15
20
25
30
35
40
45

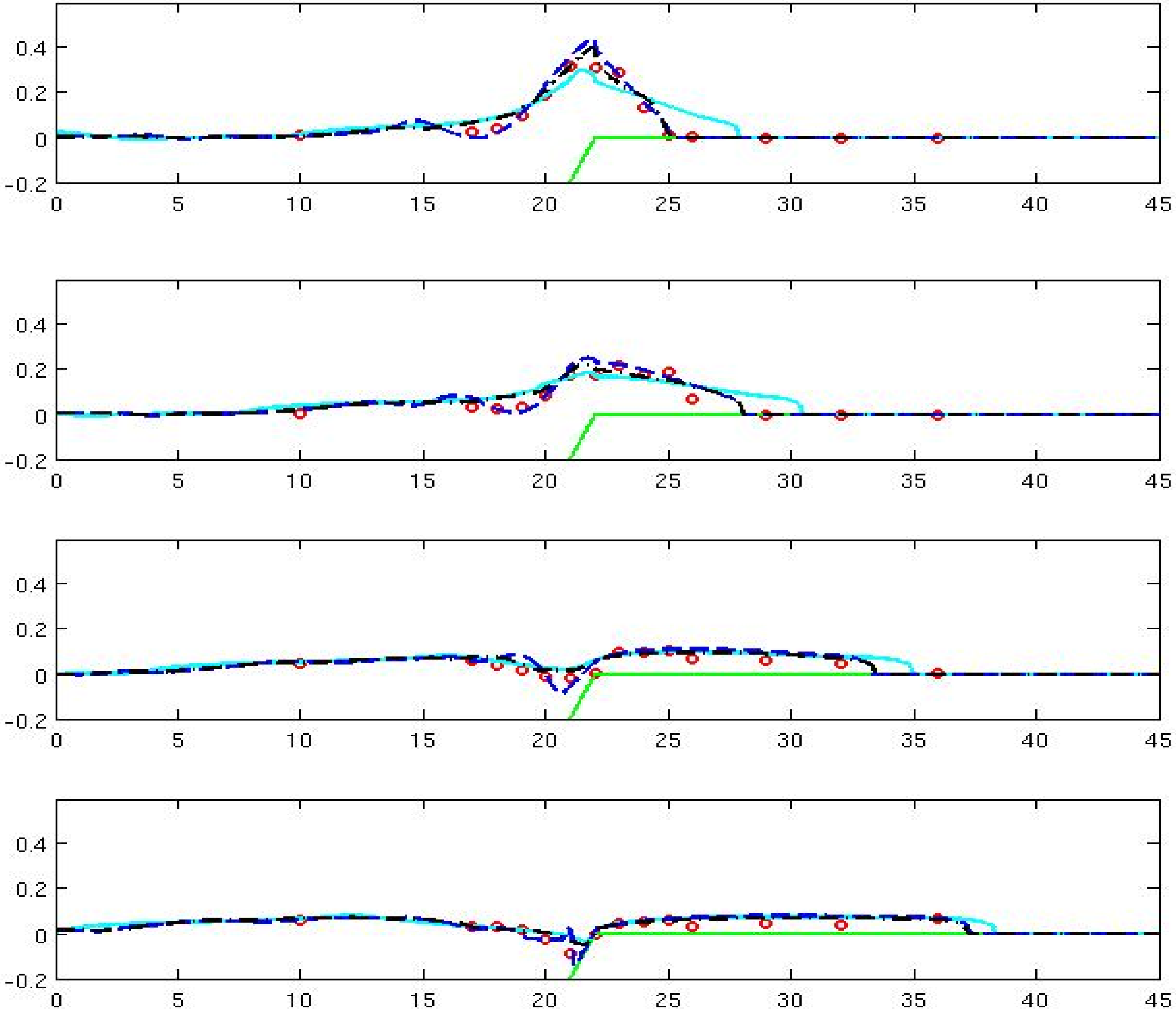
$t = 60.3$



$t = 58.3$

$t = 60.3$

$t = 64.3$

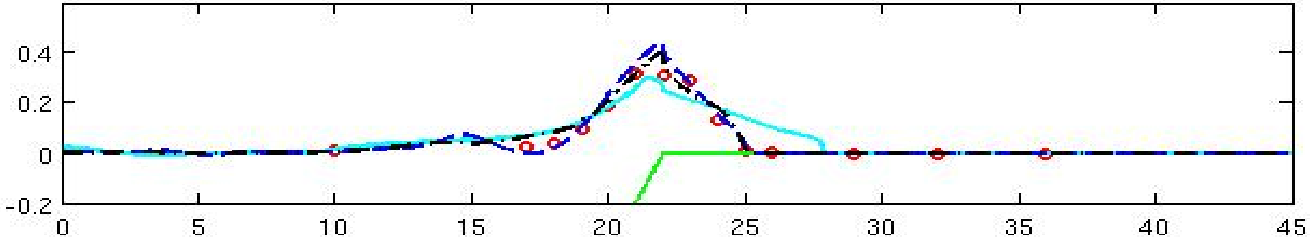


$t = 58.3$

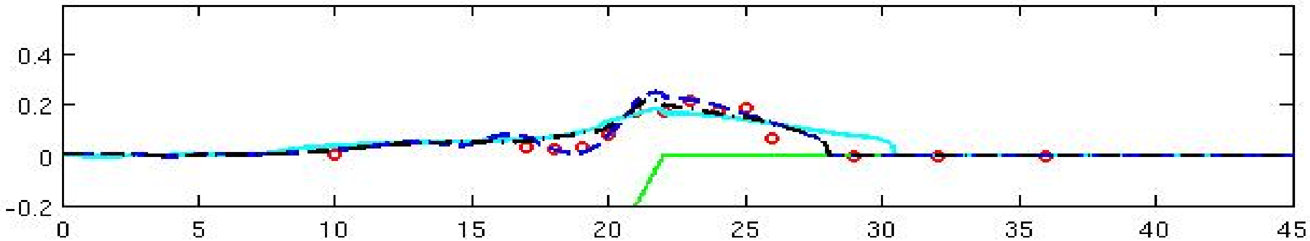
$t = 60.3$

$t = 64.3$

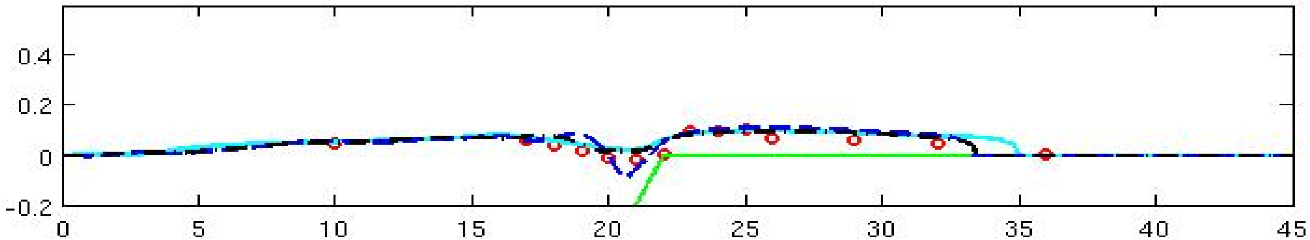
$t = 67.8$



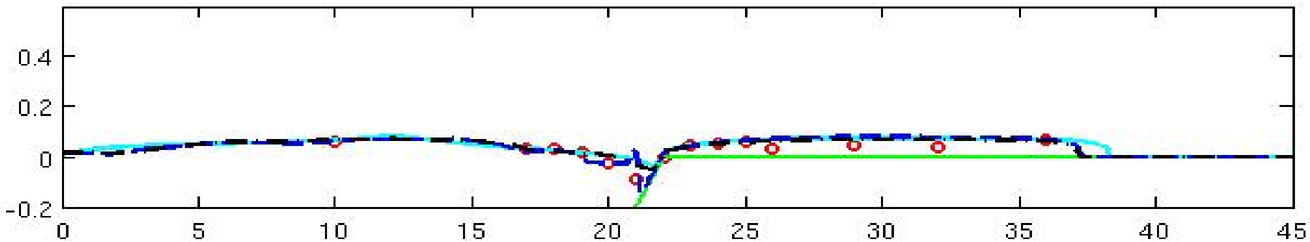
$t = 58.3$



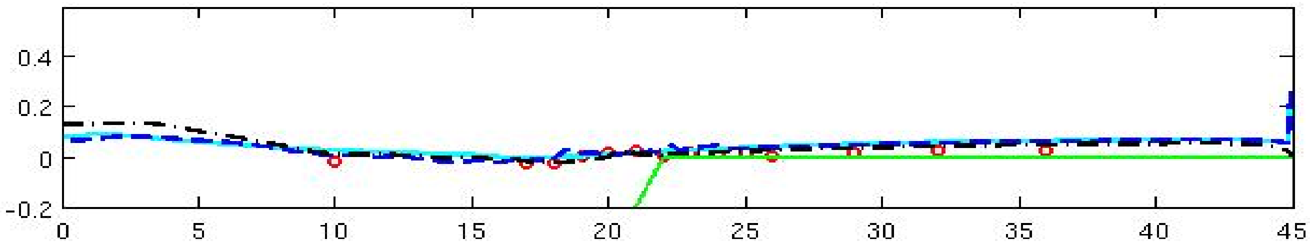
$t = 60.3$



$t = 64.3$



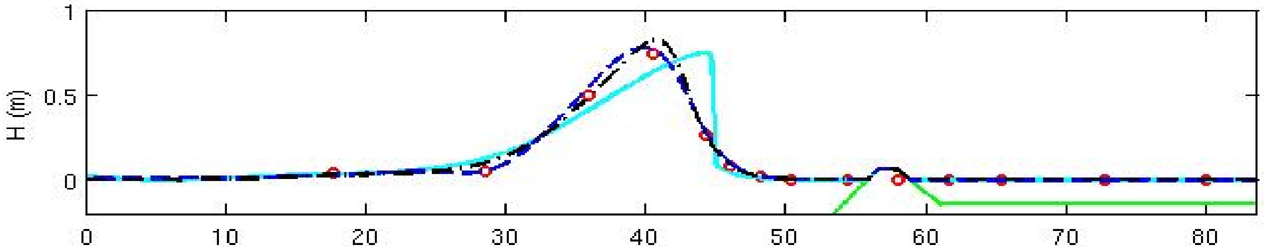
$t = 67.8$



$t = 77.3$

75-cm solitary over a reef (includes a fore reef slope of $1/12$ and a $0.2m$ reef crest.)

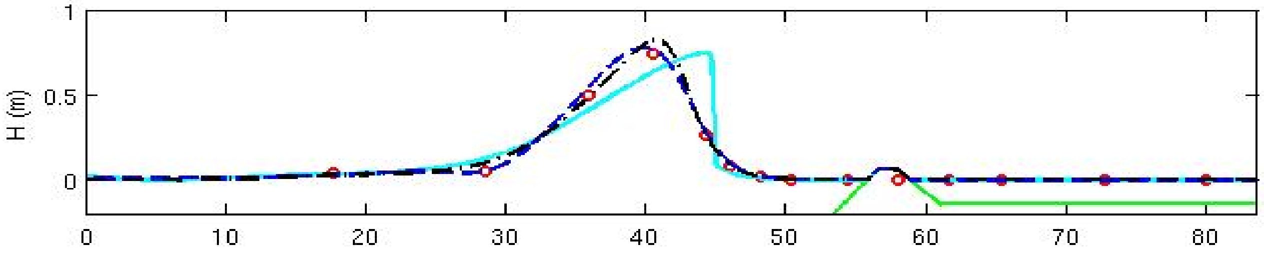
$$L = [0,\ \ 104m],\quad d = 2.5m,\quad \Delta x/d = 0.1m,\quad CFL = 0.4,\quad n_m = 0.012,\quad A/d = 0.3$$



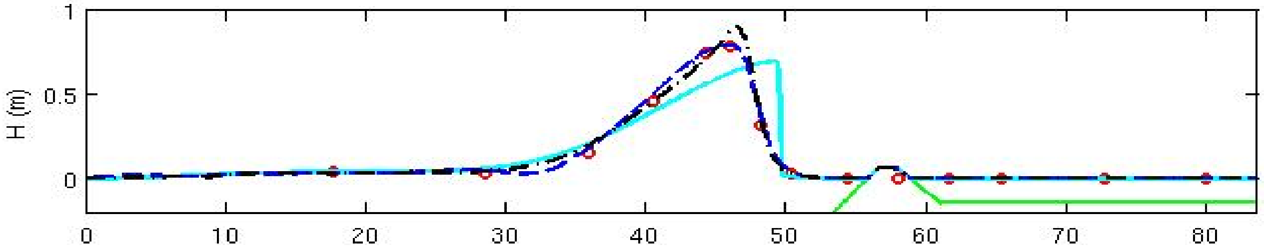
$t = 63$ Shoaling

75-cm solitary over a reef (includes a fore reef slope of $1/12$ and a $0.2m$ reef crest.)

$$L = [0, \; 104m], \quad d = 2.5m, \quad \Delta x/d = 0.1m, \quad CFL = 0.4, \quad n_m = 0.012, \quad A/d = 0.3$$



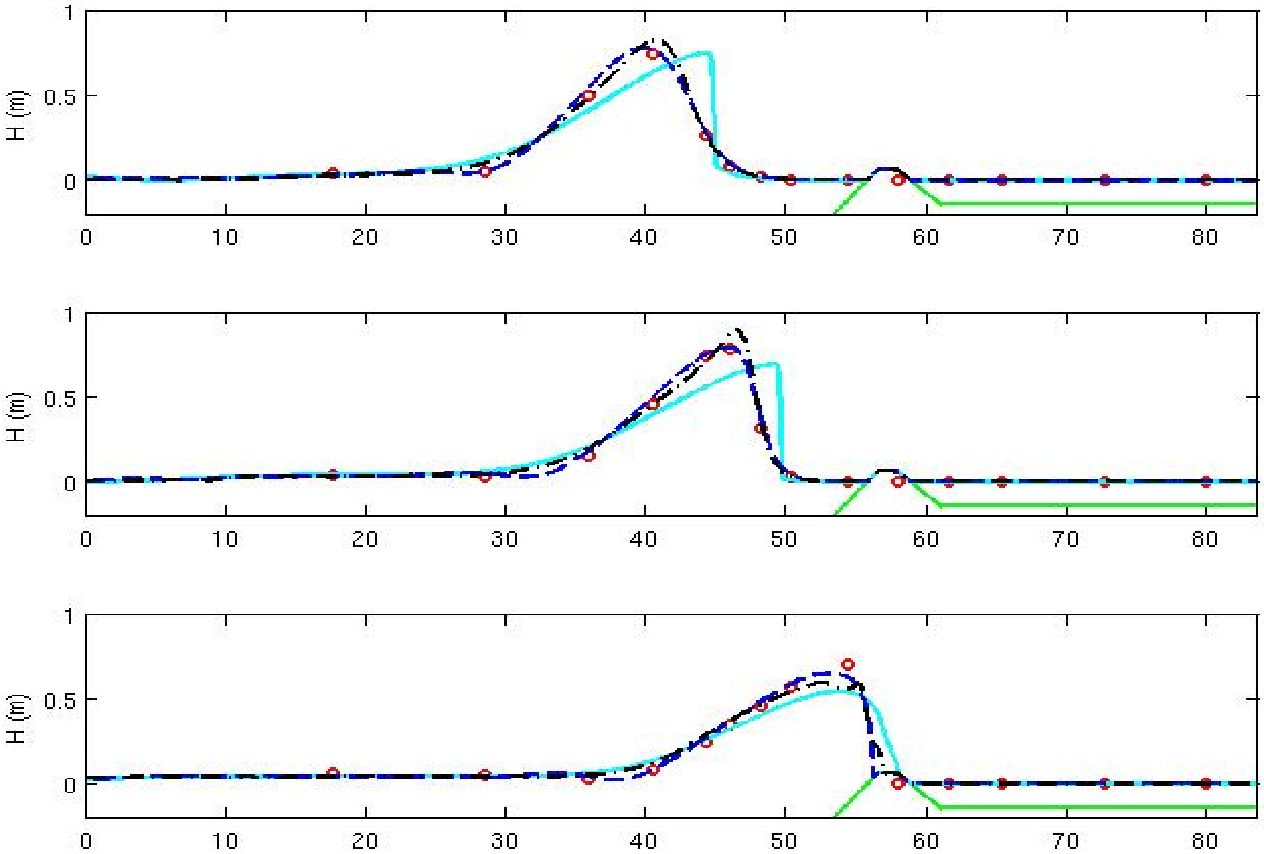
$t = 63$ Shoaling



$t = 65$ Vertical profile, before breaking

75-cm solitary over a reef (includes a fore reef slope of $1/12$ and a $0.2m$ reef crest.)

$$L = [0,\ \ 104m],\quad d = 2.5m,\quad \Delta x/d = 0.1m,\quad CFL = 0.4,\quad n_m = 0.012,\quad A/d = 0.3$$



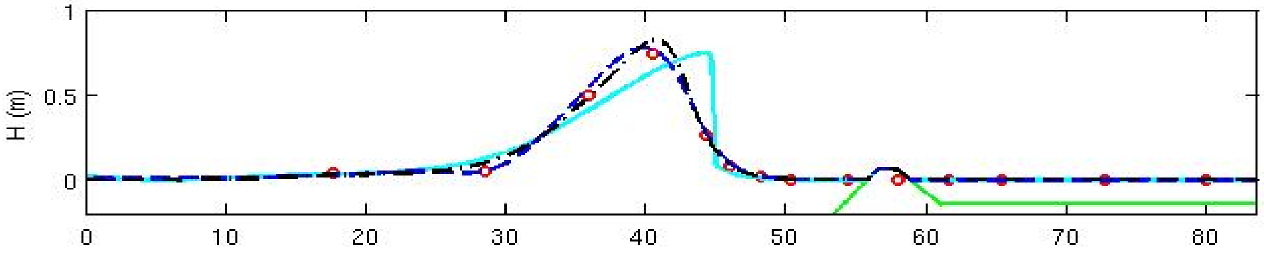
$t = 63$ Shoaling

$t = 65$ Vertical profile, before breaking

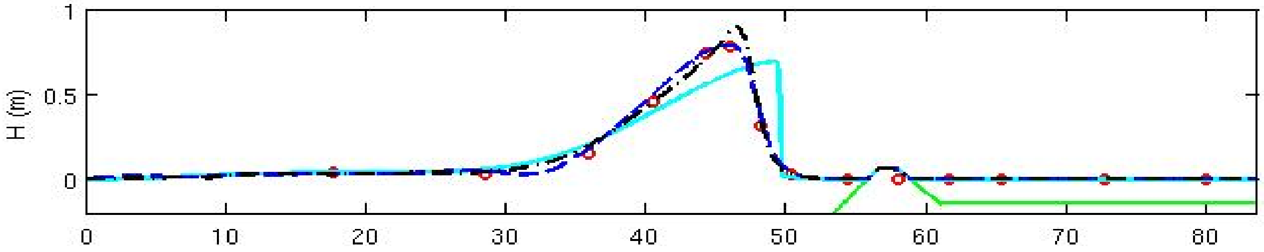
$t = 68.5$ During breaking (starts around 67)

75-cm solitary over a reef (includes a fore reef slope of $1/12$ and a $0.2m$ reef crest.)

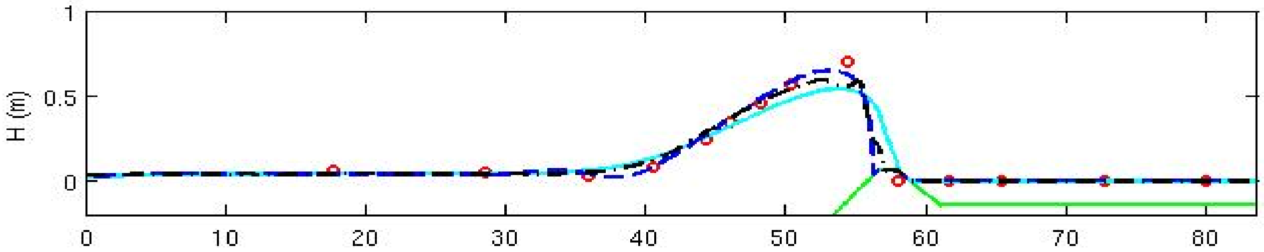
$L = [0,\ \ 104m],\quad d = 2.5m,\quad \Delta x/d = 0.1m,\quad CFL = 0.4,\quad n_m = 0.012,\quad A/d = 0.3$



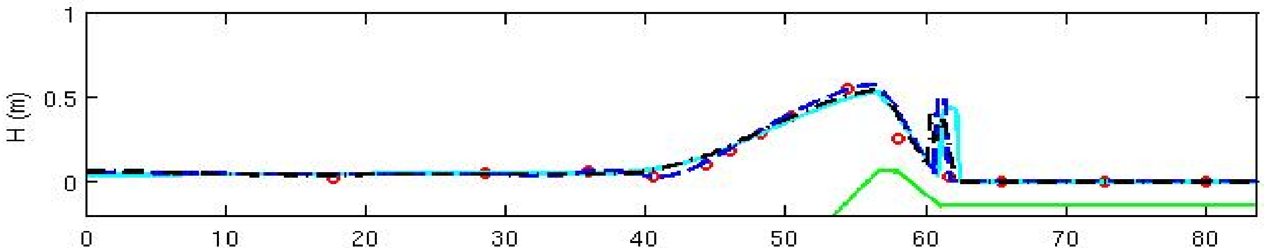
$t = 63$ Shoaling



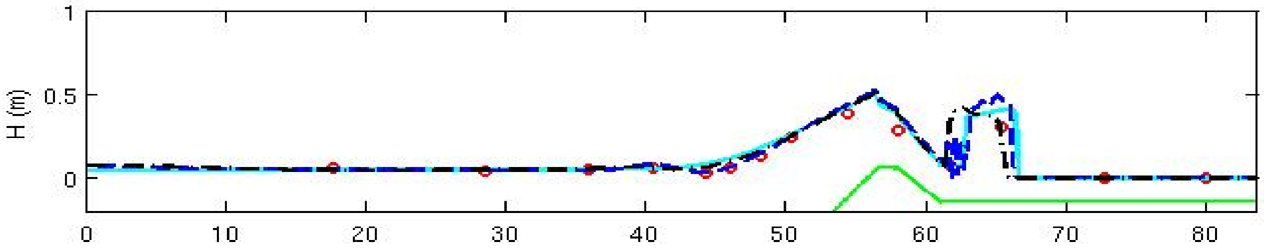
$t = 65$ Vertical profile, before breaking



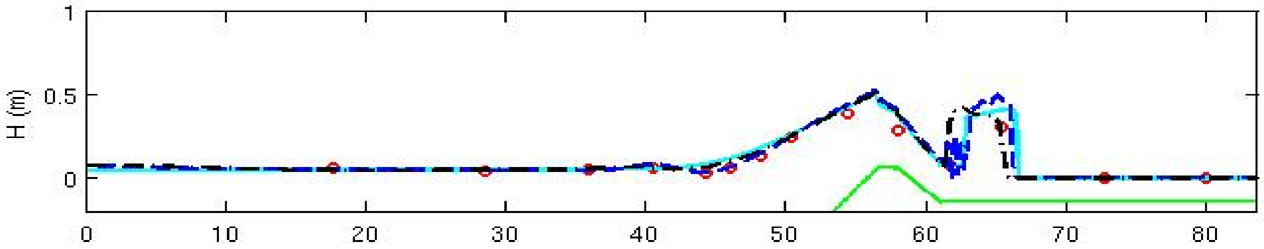
$t = 68.5$ During breaking (starts around 67)



$t = 70.5$ Wave jet hits still water



$t = 72.7$ Hydraulic jump develops



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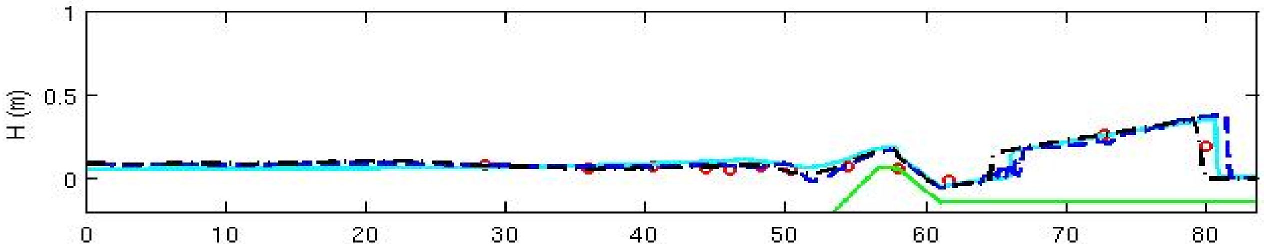
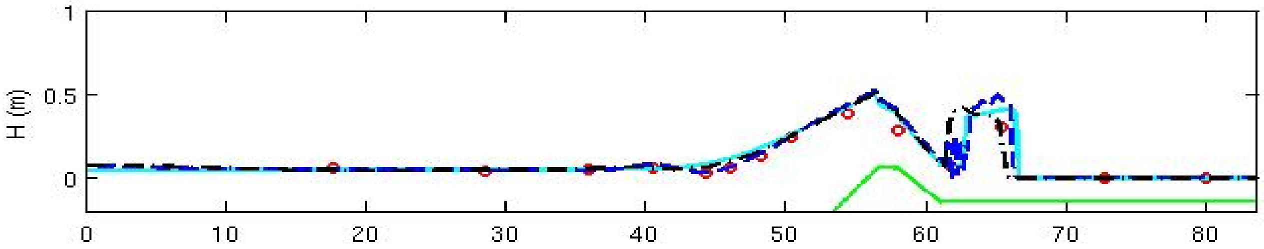


$t = 80.8$ Bore propagation



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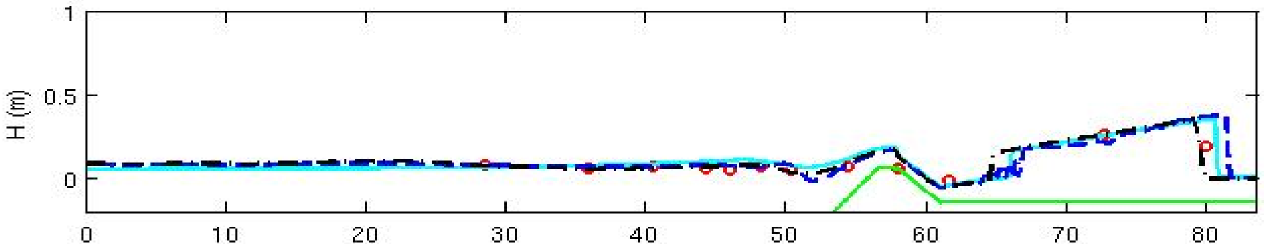
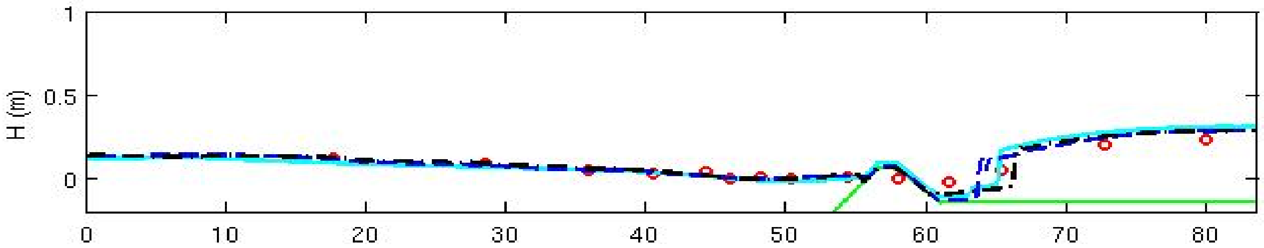
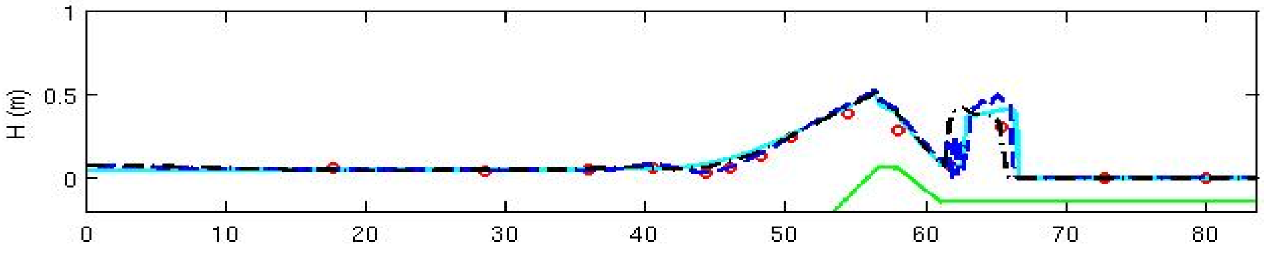


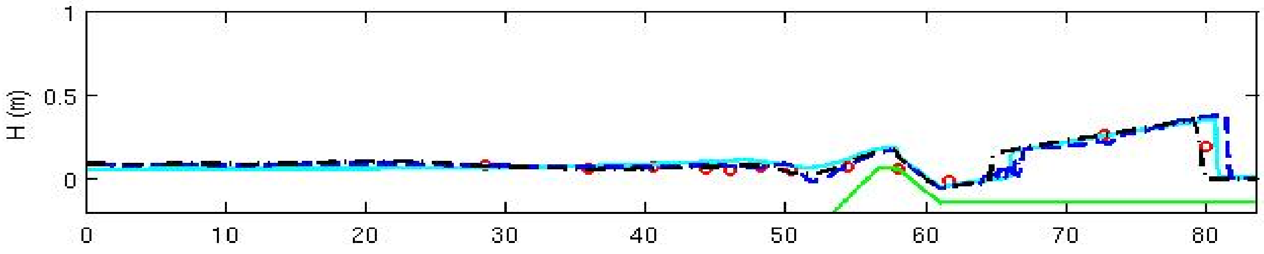
$t = 80.8$ Bore propagation



$t = 98.2$ Reflection of the bore



$t = 72.7$ Hydraulic jump develops



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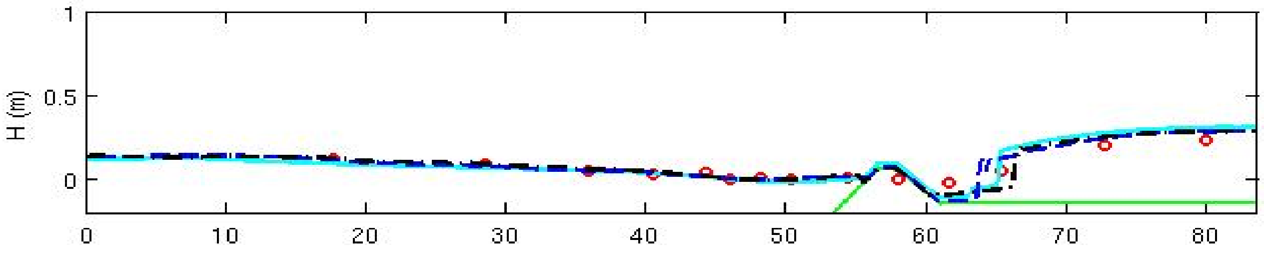

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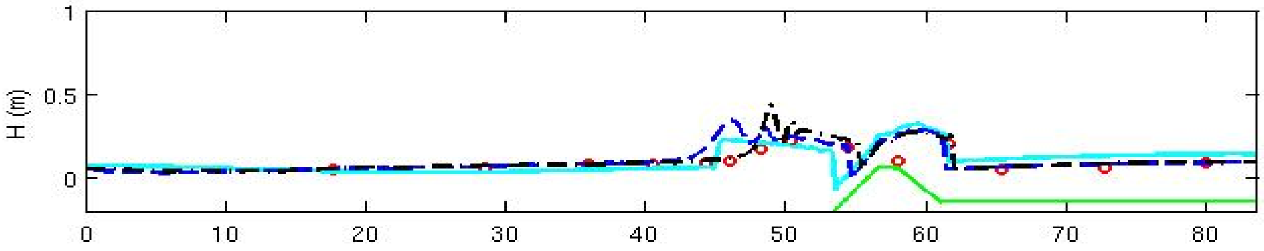
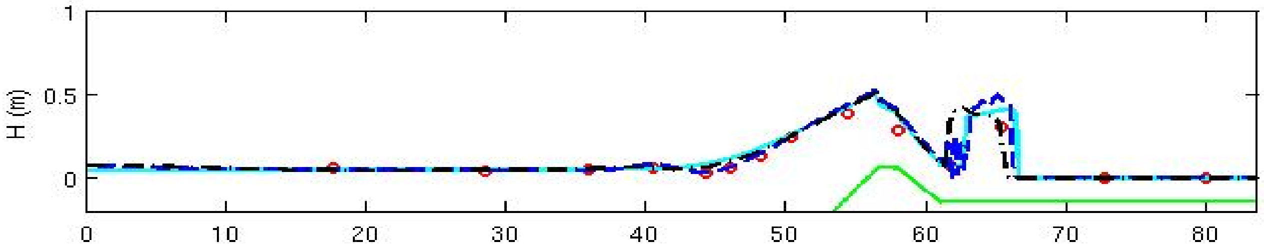
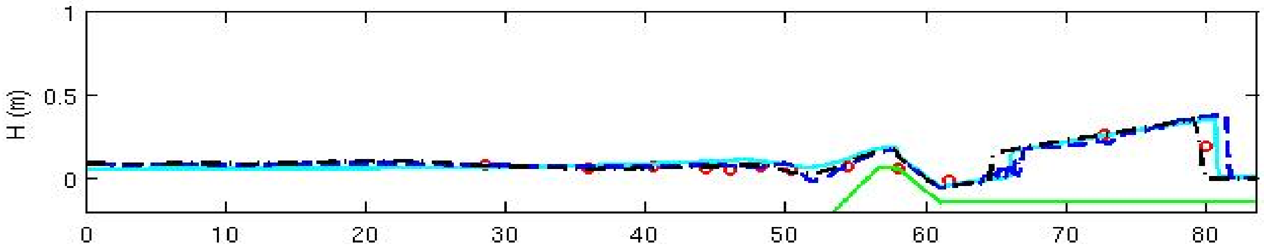


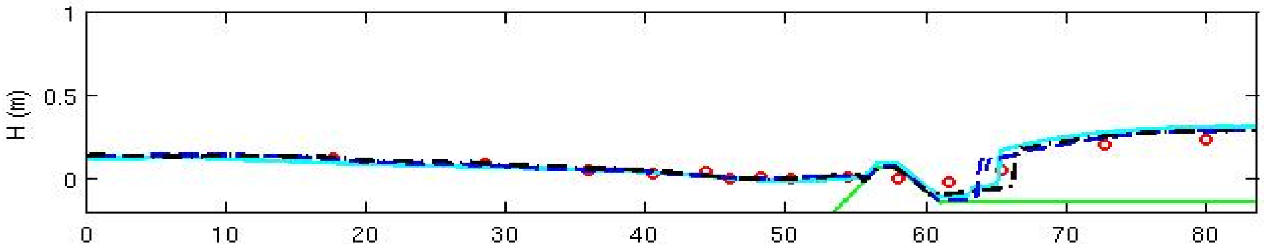
$t = 112$ Hydraulic jump on the fore reef



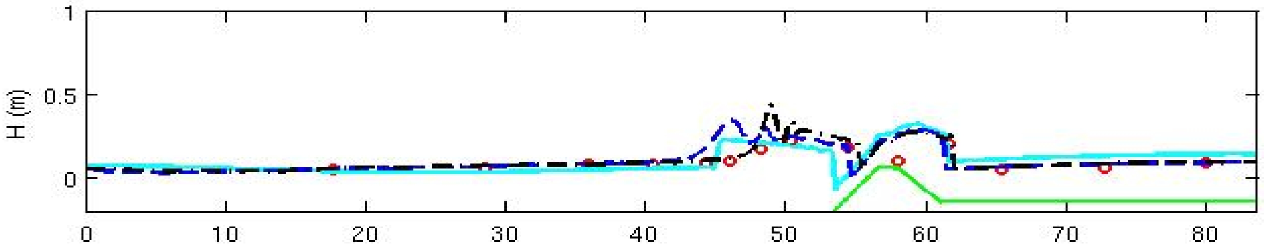
$t = 72.7$ Hydraulic jump develops



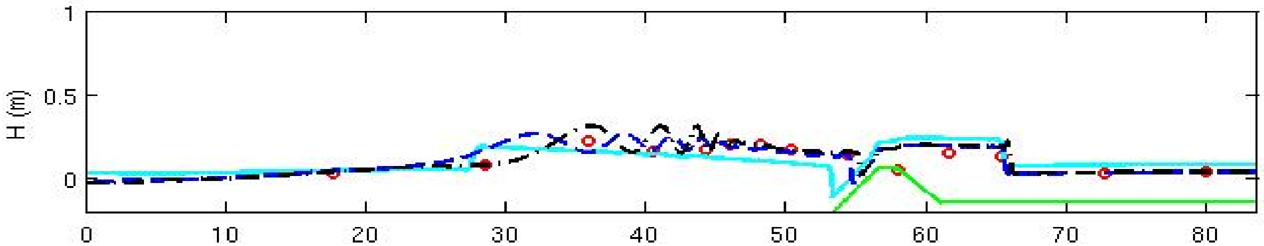
$t = 80.8$ Bore propagation



$t = 98.2$ Reflection of the bore



$t = 112$ Hydraulic jump on the fore reef



$t = 120.3$

Conclusions

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Conclusions

- A 1D alternate hybrid FV/FD conservative numerical model with shock-capturing capabilities for solving Nwogu's and MS equations, formulated in conservative form as to have identical flux terms as the SWE, has been developed.

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Ongoing work

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• **2D** extension of the present model

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Ongoing work

- **2D** extension of the present model
- Extend to other Boussinesq-type models

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