Activity Report for PADECOT–Months 6–12

I. WP2: Control and estimation of the Moskowitz model.

The Moskowitz Hamilton-Jacobi (HJ) Partial Differential Equation (PDE) is a well-know continuum model of highway traffic flow. It is the main goal of WP2 to develop boundary feedback laws as well as boundary observers for a class of (potentially viscous) HJ PDEs. The activities of WP2 are organized as follows.

A. Task 2.1: Design of a boundary feedback controller for the Moskowitz model.

Undoubtedly, maximizing the throughput at bottleneck locations is one of the most important goals of any traffic flow control algorithm. For control purposes, accurate traffic flow models may distinguish the traffic dynamics at bottlenecks from the dynamics of the rest of traffic. Typically, traffic flow control at a bottleneck area may be achieved manipulating the flow at a location upstream of the bottleneck. This location may be either fixed, such as, e.g., in the case of ramp metering or moving, such as, e.g., in the case where Variable Speed Limits (VSLs) are applied, activated at a location closer to or farther from the bottleneck location (depending, potentially, on the flow/density at the bottleneck or the VSL-application areas). Such traffic flow models may consist, in addition of a PDE state, of an ODE state that describes the traffic dynamics at the bottleneck area. Motivated by the need for control at bottleneck areas and capitalizing on the close relation of the cumulative number-of-vehicles equation, in particular, the Moskowitz model, to the conservation-of-vehicles equation, in particular, the Lighthill-Whitham-Richards (LWR) model (the cumulative number of vehicles may be derived from the density of vehicles via a certain integral transformation and, respectively, the density of vehicles may be obtained from the cumulative number of vehicles via differentiation), the following activities are performed.

A1. A methodology for stabilization of general nonlinear systems with actuator dynamics governed by general, quasilinear, first-order hyperbolic PDEs is presented. Since for such PDE-ODE cascades the speed of propagation depends on the PDE state itself (which implies that the prediction horizon cannot be a priori known analytically), the key design challenge is the determination of the predictor state. This challenge is resolved and a PDE predictor-feedback control law that compensates the transport actuator dynamics is introduced. Since it may be intriguing as to what is the exact relation of the cascade to a system with input delay, the fact that the considered PDE-ODE cascade gives rise to a system with input delay, with a delay that depends on past input values (defined implicitly via a nonlinear equation), is highlighted.

A2. The stabilization problem of a transport PDE/nonlinear ODE cascade, in which the PDE state evolves on a domain whose length depends on the boundary values of the PDE state itself, is introduced and solved. In particular, a predictor-feedback control design, which compensates such transport PDE dynamics, is developed. The relation of the PDE-ODE cascade to a nonlinear system with input delay that depends on past input values is also highlighted and the predictor-feedback control design for this representation is presented as well.

B. Task 2.2: Observer design for estimation of the PDE state assuming boundary measurements.

the availability of real-time traffic state estimates is a prerequisite for successful application of traffic management and control strategies. In particular, lane-specific highway traffic management and control has considerable potential in traffic flow optimization. However, the effectiveness of lane-based traffic management and control strategies largely depends on the quality and accuracy of traffic monitoring at

Further information can be found here.

a lane level. A reliable and cost-effective traffic monitoring solution should amply rely on the presence of connected vehicles, which are capable of providing accurate position and speed information (they can act as mobile sensors), whereas it should rely on the minimum number (e.g., at highway boundaries) of (costly) fixed detectors. For these reasons, we address the problem of per-lane density estimation as well as ramp flow estimation in highways, via the development of a model-based estimation approach, which relies largely on the presence of connected vehicles.

Capitalizing on the close relation of the cumulative number-of-vehicles equation (in particular, the Moskowitz model) to the conservation-of-vehicles equation (in particular, the state of the cumulative number-of-vehicles model may be obtained from the state of the conservation law model) and in order to obtain an estimation methodology that is as ready as possible for real data testing and even for potential actual implementation, we develop a methodology based on the following three basic ingredients: (1) a data-driven version of the conservation-of-vehicles equation (in its time- and space-discretized form); (2) the utilization of position and speed information from connected vehicles reports, as well as total flow measurements obtained from a minimum number (sufficient for the observability of the model) of fixed detectors, such as, for example, at the main entry and exit of a given highway stretch; and (3) the employment of a standard Kalman filter. Furthermore, necessary and sufficient conditions for the (strong) structural observability of the introduced model are established (properties, which are rarely studied in the literature on traffic estimation), which yield the fixed detectors requirements needed for the proper operation of the developed estimation scheme.

C. Task 2.3: Combination of the full-state feedback controller with the observer in a dynamic output-feedback control design.

Arguably, it is desirable that any real-time traffic flow control/estimation algorithm, besides being able to efficiently exploit the capabilities of the available actuators and sensors, potentially, by also capitalizing on appropriate coordination and integration of different strategies, can be also made robust with respect to sensor or actuator errors and with respect to noisy measurements (which may deteriorate the performance of real-time traffic flow control).

We are concerned with the coordinated, fault-tolerant, and noise-robust highway traffic flow planning, control, and estimation. The control strategies, which may be employed via ramp metering or variable speed limits (or, via combination of both), utilize efficiently the typical minimum actuation capabilities, as they are constructed considering “bilateral boundary actuation, i.e., at the two boundary ends of a given highway stretch. The “dual estimation strategies, which may be employed combining measurements from fixed detectors and probe vehicles, exploit in an efficient way the typically available measurement information from fixed detectors, as the basis for their design is a bilateral sensing framework, in which measurements from the two highway boundaries are appropriately combined. The control and estimation schemes are developed for a class of viscous Hamilton-Jacobi PDEs, which include the Moskowitz model with quadratic Hamiltonian as special case, and are capable of capturing certain realistic “smoothing effects in traffic flow due to drivers look-ahead ability.

The output-feedback control design consists of three ingredients. The first, is the design of the feedforward actions at both boundaries, which provide the desired trajectory for the traffic state. This is achieved via solving the nonlinear trajectory generation problem for this type of PDEs. The second ingredient is the development of full-state feedback laws for the two boundary ends of the highway, which guarantee tracking of the desired trajectory of the traffic flow with an arbitrary decay rate. The third element is the construction of a nonlinear observer for estimation of the traffic state via utilization of measurements from both highway boundaries. The three different elements are then combined together in a complete observer-based output-feedback traffic flow control strategy. All of the designs are constructed interlacing PDE feedback linearization with PDE backstepping.

II. WP4: Analysis

A. Task 4.1: Stability analysis of the Moskowitz model under the proposed controller with the construction of a Lyapunov function.

A1. For the class of systems considered in Task 2.1.A1 of WP2, local asymptotic stability of the closed-loop system, under the developed feedback law, in the C1 norm of the PDE state is established as follows. Due to the potential formation of shock waves in the solutions of quasilinear, first-order hyperbolic PDEs (which is related to the fundamental restriction for systems with time-varying delays that the delay rate is bounded by unity), one should limit herself/himself to a certain feasibility region around the origin. It is then shown that the PDE predictor-feedback law achieves asymptotic stability of the closed-loop system and an estimate of its region of attraction is provided. The analysis combines Lyapunov-like arguments and input-to-state stability (ISS) estimates.

A2. For the class of systems considered in Task 2.1.A2 of WP2, the closed-loop system, under the developed predictor-feedback control law, is shown to be locally asymptotically stable, in the C1 norm of the PDE state, via the employment of a Lyapunov-like argument and the introduction of a backstepping transformation. The stability result is local due to an inherent limitation of the class of transport PDEs under consideration, which ensures the well-posedness of the given transport PDE. More specifically, this restriction guarantees that, in an equivalent formulation of the transport PDE that employs a constant PDE domain and a transport speed that depends on the boundary valuesof the PDE state as well as its first-order spatial derivative, the transport speed remains always strictly positive as well as uniformly bounded from above and below by finite constants.

A3. For the model introduced in Task 2.2.A we study its observability properties. Specifically, we provide sufficient and necessary conditions for the structural observability as well as the strong structural observability of the model, adopting a graph-theoretic approach; it should be noted that observability properties are rarely studied in the literature on traffic estimation. As a result, we characterize the fixed detectors configuration that guarantee the proper operation of the developed traffic state estimation scheme. The stability properties of the estimator are also discussed.

A4. All of the designs in Task 2.3.A are explicit since they are constructed interlacing a feedback linearizing transformation (which is introduced and which is inspired by the so-called Hopf-Cole transformation) with PDE backstepping. Due to the fact that the linearizing transformation is locally invertible, only regional stability results are established, which are, nevertheless, accompanied with region of attraction estimates. Our stability proofs are based on the utilization of the linearizing transformation together with the employment of backstepping transformations, suitably formulated to handle the case of bilateral actuation and sensing.

Specifically, we first establish the well-posedness of the feedforward controllers for the original nonlinear PDE system, for reference outputs that belong to Gevrey class (of certain order) with sufficiently small magnitude, via the employment of the feedback linearizing transformation, which allows us to convert the original nonlinear problem to a motion planning problem for a linear heat equation. Second, we establish local asymptotic stability of the closed-loop system (via the utilization of a modified version of the feedback linearizing transformation), under the full-state feedback laws, in H^1 norm, employing a Lyapunov functional and we provide an estimate of the region of attraction of the controllers. Our stability result is local in H^1 norm due to the fact that the linearizing transformation is invertible only locally and, in particularly, the size of the supremum norm of the transformed PDE state should be appropriately restricted. Finally, we show that the bilateral, observer-based output-feedback controller achieves local asymptotic stabilization of the reference trajectory in H^1 norm.

III. WP6: Communication, dissemination, and exploitation

The activities in WP6 may be divided in the following tasks

Task 6.1: The dissemination plan includes, among others, paper publications, conference/seminar presentations, workshop/symposium organization, and webpage creation.

Task 6.2: The exploitation plan includes, among other things, definition of new research directions, industry application, proposal writing, and advertisement of the results.

Task 6.3: The communication and public engagement plans include video creation and dissemination, newspaper/radio interviews, as well as specific outreach activities,

which include, among other things, publications (see Section III-A below) and specific events (see Section III-B below as well as here for photos, videos, and announcements):

A. Papers

a) Journal:

5. N. Bekiaris-Liberis and R. Vazquez, "Nonlinear bilateral output-feedback control for a class of viscous Hamilton-Jacobi PDEs," *Automatica*, under review, 2018.
4. I. Karafyllis, N. Bekiaris-Liberis, and M. Papageorgiou, "Feedback control of nonlinear hyperbolic PDE systems inspired by traffic flow models," *IEEE Transactions on Automatic Control*, under review, 2018.
3. N. Bekiaris-Liberis and M. Krstic, "Compensation of transport actuator dynamics with input-dependent moving controlled boundary," *IEEE Transactions on Automatic Control*, to appear, 2018.
2. N. Bekiaris-Liberis and M. Krstic, "Compensation of actuator dynamics governed by quasilinear hyperbolic PDEs," *Automatica*, vol. 92, pp. 29–40, 2018.
1. N. Bekiaris-Liberis, C. Roncoli, and M. Papageorgiou, "Highway traffic state estimation per lane in the presence of connected vehicles," *Transportation Research Part B*, vol. 106, pp. 1–28, 2017.

b) Conference:

5. N. Bekiaris-Liberis and R. Vazquez, "Nonlinear bilateral full-state feedback trajectory tracking for a class of viscous Hamilton-Jacobi PDEs," *IEEE Conference on Decision and Control*, submitted, 2018.
4. I. Karafyllis, N. Bekiaris-Liberis, and M. Papageorgiou, "Traffic flow inspired analysis and boundary control for a class of 2×2 hyperbolic systems," *European Control Conference*, 2018.
3. N. Bekiaris-Liberis and M. Krstic, "Control of nonlinear systems with actuator dynamics governed by quasilinear first-order hyperbolic PDEs," *European Control Conference*, 2018.
2. N. Bekiaris-Liberis and M. Krstic, "Compensation of transport actuator dynamics with input-dependent moving controlled boundary," *European Control Conference*, 2018.
1. N. Bekiaris-Liberis, C. Roncoli, and M. Papageorgiou, "Predictor-based adaptive cruise control design with integral action," *IFAC Symposium on Control in Transportation Systems*, 2018.

B. Specific Events

1 Dr. Bekiaris-Liberis delivered a presentation on *Partial differential equation model-based control of traffic flow*, at the Dynamic Systems & Simulation Laboratory, Department of Production Engineering& Management, Technical University of Crete, Greece, on October 06, 2017.

2. Dr. Bekiaris-Liberis delivered a seminar on *Control of transport PDE-ODE cascades*, in the department of Engineering Science, University of Oxford, Oxford, UK, on October 16, 2017.
3. Dr. Bekiaris-Liberis delivered a seminar on *Control of transport PDE-ODE cascades*, in the department of Mathematics, National Technical University of Athens, Athens, Greece, on November 03, 2017.
4. Prof. Dimos Dimarogonas, Department of Automatic Control, School of Electrical Engineering, KTH Royal Institute of Technology, delivered a seminar on *Distributed hybrid control of multi-agent systems under high level specifications*, at Technical University of Crete, Greece on December 08, 2017.

5. Dr. Bekiaris-Liberis delivered a seminar on *Control of PDE-ODE interconnections for traffic and energy systems*, in the department of Electrical & Computer Engineering, Technical University of Crete, Greece on January 18, 2018.
6. Dr. Bekiaris-Liberis delivered a presentation on *PDE-based traffic flow control at distant bottlenecks*, at the Dynamic Systems & Simulation Laboratory, Department of Production Engineering & Management, Technical University of Crete, Greece, on January 26, 2018.
7. Dr. Bekiaris-Liberis delivered a seminar on *Nonlinear control of transport PDE-ODE interconnections*, in the department of Electrical & Computer Engineering, Aristotle University of Thessaloniki, Greece on February 23, 2018.
8. Prof. Rafael Vazquez, Department of Aerospace Engineering, University of Seville, delivered a seminar on *Backstepping Control of PDEs: Basic Theory and Some Recent Results*, at Technical University of Crete, Greece on March 05, 2018.
9. Dr. Bekiaris-Liberis delivered a presentation on *Nonlinear control of transport PDE-ODE interconnections*, at the College of Mathematics, Physics, and Information Engineering, Zhejiang Normal University, Jinhua, China, on March 19, 2018.
10. Dr. Bekiaris-Liberis delivered a presentation on *Nonlinear Control of Transport PDE-ODE Interconnections*, at the College of Control Science & Engineering, Zhejiang University, Hangzhou, China, on March 21, 2018.
11. Dr. Bekiaris-Liberis delivered a presentation on *Nonlinear Control of Transport PDE-ODE Interconnections*, at the College of Information Science & Technology, Donghua University, Shanghai, China, on March 23, 2018.
12. Dr. Bekiaris-Liberis participated and delivered a presentation on *Traffic Control*, in the Meeting with the Technology Day (open day for high-school students), at Technical University of Crete, Greece, on March 26, 2018.

IV. Other Activities

1. Attendance of the online course from the European Patent Academy entitled “From research to business”.
2. Associate editor for the 14th Workshop on Time Delay Systems,
3. Member of the Scientific Committee for the 2nd Symposium on Management of Future Motorway and Urban Traffic Systems.