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Nonlinear predictor-feedback cooperative adaptive cruise control of vehicles with nonlinear dynamics and input delay

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Summary

We construct a nonlinear predictor-feedback cooperative adaptive cruise control (CACC) design for homogeneous vehicular platoons subject to actuators delays, which achieves: (i) positivity of vehicles' speed and spacing states, (ii) $\mathcal{L}_\infty$ string stability of the platoon, (iii) stability of each individual vehicular system, and (iv) regulation to the desired reference speed (dictated by the leading vehicle) and spacing. The design relies on a nominal, nonlinear adaptive cruise control (ACC) law that we construct, which guarantees (i)–(iv) in the absence of actuator delay, and nonlinear predictor feedback. We consider a classical (for ACC/CACC design) third-order, nonlinear model subject to input delay, for the vehicles' dynamics. The proofs of the theoretical guarantees (i)–(iv) rely on derivation of explicit estimates on solutions (both during open-loop and closed-loop operation), capitalizing on the ability of predictor feedback to guarantee complete delay compensation after the dead-time interval has elapsed, and derivation of explicit conditions on initial conditions and parameters of the nominal control law. We also present consistent simulation results, considering a platoon of ten vehicles, which validate the design developed.

KEYWORDS

delay compensation, nonlinear cooperative adaptive cruise control, nonlinear predictor feedback, safety of vehicular platoons, string stability of vehicular platoons

1 | INTRODUCTION

Delay compensation in vehicular platoons equipped with ACC and CACC capabilities is a significant objective of ACC/CACC designs, in view of its potential in improvement of the safety and performance properties of platoons.[1–10] Three different types of delays are, typically, evident in such systems, namely, actuation, sensing, and communication delays. Each delay type may have a negative effect in individual vehicle stability and string stability, when is left uncompensated; while each of delay type requires specific treatment for its compensation. Here we address actuation delay, which, typically, takes the largest values among the three types.[10]

ACC and CACC designs, aiming at delay compensation* in vehicular platoons, have been already developed.[1,2,6,9–18] Almost all of these results utilize linear or linearized models for each individual vehicular system, for either control design or stability and string stability analysis. Nevertheless, these models, in certain scenarios, may not be as realistic as nonlinear vehicle models (for both control design and analysis) that may capture additional, lower-level vehicle dynamics.[19–23] Such linear vehicle models have been successfully utilized in these works for delay-compensating ACC/CACC design, with derivation of theoretical guarantees and validation in experimental platforms. However, considering nonlinear models of vehicle dynamics is important because a nominal (i.e., without a predictor structure), feedback linearizing pre-compensator, which may be implicitly employed in control of vehicular platoons (to subsequently enabling linear, ACC/CACC designs), may not result in a linear vehicle model, due to the presence of actuation delay. To the best of our knowledge, the only result that is related to construction of a nonlinear predictor-feedback ACC (or CACC) design can be found in the work of Molnar et al.[13]

In the present paper, complementing the results in Molnar et al.,[13] we (a) consider a third-order model for the vehicles' dynamics; (b) construct a new nonlinear ACC law in the case in which there is no input delay; (c) design a predictor-feedback CACC law relying on real-time measurements of the acceleration and control input of the preceding vehicle (thus avoiding utilization of open-loop predictors, for the preceding vehicle's states, which, potentially, may be less robust); (d) provide the predictors formulae as explicitly as possible; (e) consider a platoon of vehicles; and (f) provide explicit conditions on initial conditions and control parameters, which guarantee positivity of speed and spacing states, as well as stability and regulation. In particular, we construct a nonlinear predictor-feedback CACC law, which aims at actuation delay compensation for vehicular platoons in which each vehicle's dynamics are described by a third-order, nonlinear system with input delay. The design relies on two ingredients-a nominal (for the delay-free case) nonlinear ACC design of constant time headway (CTH) type and states' predictors. The nominal ACC law is constructed utilizing the design procedure developed by Krstic and Bement (for strict-feedback nonlinear systems).[24] Note that although the underlying, ACC design in the delay-free case is inspired from the work of Krstic and Bement,[24] contributions (a)–(f) described above are novel. The reason is that the systems considered here involve input delay and describe the dynamics of platoons of connected/automated vehicles. In more detail, due to the presence of input delay, we introduce a novel predictor-feedback CACC design and a new analysis strategy for establishing stability and positivity of speed/spacing states. While due to the fact that the system considered may describe the dynamics of vehicular platoons, we introduce a novel string stability analysis strategy and a new, proper extension (see also the discussion in the paragraph below (6) for more technical details) of the underlying, delay-free ACC law, for making it suitable for control of platoons of connected/automated vehicles.

In fact, because to predict the speed of the preceding vehicle (employed in the nominal ACC design), measurements of the control input variable and acceleration of the preceding vehicle are required (obtained via vehicle-to-vehicle communication), the resulting control law is of CACC type. Note that, differently from existing results on CACC, information of the state and control input from only the preceding vehicle is required, for control implementation in the ego vehicle. This minimum V2V (vehicle-to-vehicle) communication requirement also implies that the control strategy developed here could be implemented in practical scenarios in which a single ego vehicle, which is connected and automated, follows a vehicle that may be only connected, that is, able to transmit information via V2V communication, but not necessarily automated.

The feedback law constructed guarantees the primary objectives of a CACC design, namely, (i) positivity of speed and spacing states, (ii) $\mathcal{L}_\infty$ string stability, (iii) stability of each individual vehicular system, and (iv) regulation to the desired reference speed (dictated by the leading vehicle) and spacing. The proofs of guarantees (i)–(iv) rely on derivation of explicit estimates on solutions, capitalizing on the ability of predictor feedback to achieve complete delay compensation after the dead-time interval has elapsed, and derivation of explicit conditions on initial conditions and parameters of the nominal controller. The conditions derived on the initial states are consistent with the practical requirement that there is no finite-escape time phenomenon appearing and that the speed/spacing states remain positive during open-loop operation (i.e., during the dead-time interval). The conditions on the control law parameters are consistent with the requirements needed to guarantee individual vehicle stability, regulation to the desired speed/spacing, as well as positivity of speed/spacing states during closed-loop operation, in the nominal, delay-free case. While $\mathcal{L}_\infty$ string stability is guaranteed by the specific structure of the nominal, nonlinear ACC law. No restriction on the delay size or the desired time headway are imposed, which is consistent with the fact that predictor feedback guarantees that, in closed loop, each individual vehicular system inherits the properties of the respective, nominal (for the delay-free case) closed-loop system. We also demonstrate the effectiveness of the design in simulation, considering a realistic scenario in which a vehicle cuts in a platoon of nine vehicles (e.g., as result of lane changing) and it subsequently performs an acceleration/deceleration maneuver. While, for the same scenario, we further illustrate the robustness properties of the design developed to delay uncertainties.

We start in Section 2 presenting the third-order, nonlinear model of the vehicles' dynamics, together with the predictor-feedback CACC design. In Section 3 we state the main result of the paper, establishing properties (i)–(iv) for the platoon, under the CACC law developed. We validate the design in simulation in Section 4 and provide concluding remarks in Section 5. The proof of the main result is presented in Appendix A.

2 | NONLINEAR PREDICTOR-FEEDBACK CACC FOR HOMOGENEOUS PLATOONS WITH ACTUATOR DELAY

2.1 | Vehicle dynamics

We consider a homogeneous platoon of vehicles as shown in Figure 1.

Each vehicle's dynamics are modeled by the following third-order, nonlinear system, with input delay

$$\dot{s}_i(t) = v_{i-1}(t) - v_i(t), \tag{1}$$
$$\dot{v}_i(t) = -dv_i(t)^2 - g + cT_i(t) \tag{2}$$
$$\dot{T}_i(t) = -\frac{1}{\tau}T_i(t) + \frac{1}{\tau}u_i(t-D), \tag{3}$$

$i = 1, \ldots, N$, where s_i is spacing between vehicles i and $i-1$, v_i is vehicle's speed, T_i is engine torque, u_i is the individual vehicle's control variable, $D \geq 0$ is actuator delay, $t \geq 0$ is time, and d, g, c, τ are positive coefficients depending on vehicle's characteristics.[19–23,25] Vehicle dynamics (1)–(3) are considered as sufficiently reasonable for the purpose of illustrating the delay compensation benefits of predictor feedback to individual vehicle stability and string stability, as well as to safety of CACC platoons. In fact, such dynamic models are also viewed as extensions, of (classical) nonlinear vehicle models employed for ACC/CACC design and analysis,[19,21–23] to incorporate input delay, which may be more realistic in practice.[13,26,27]

For the leading vehicle's speed dynamics, adopting the notation $v_l \equiv v_0$, $T_l \equiv T_0$, $u_l \equiv u_0$, we assume that $\dot{v}_l(t) = T_l(t)$, $\dot{T}_l(t) = -\frac{1}{\tau}T_l(t) + \frac{1}{\tau}u_l(t-D)$, where u_l acts to the platoon as exogenous input. We consider such dynamics for the leading vehicle for simplicity as v_l is viewed here more as a reference input, rather than as a state that has to be regulated. However, there is no conceptual obstacle to re-design the predictor-feedback CACC law to account for different dynamics for the speed of the leading vehicle (in particular, being identical to (2)), since the predictor states, which rely on the vehicles' model, could be straightforwardly modified accordingly. This is the case as long as the control input of the leading vehicle is subject to an input delay D. Note that a uniform equilibrium point of systems (1)–(3) is obtained when all vehicles have the same, constant speed, dictated by a constant, leader's speed, say v^*, corresponding to a constant control input value $u^* = \frac{g+v^{*2}}{c}$ (and with $u^* = T^*$).

2.2 | Delay-free control design

In the delay-free case, we seek for an ACC law of the form $u_{i,nom} = f(s_i, v_i, v_{i-1}, T_i)$. For implementation of such an ACC law measurements of states s_i, v_i, v_{i-1}, and T_i are required, which can be obtained from on-board sensors. Without actuator delay, in the present paper, we construct the following, nominal feedback laws of ACC type for $i = 1, \ldots, N$

$$u_{i,nom}(t) = \frac{\tau}{c}\bar{u}_{i,nom}(t) \tag{4}$$

$$\bar{u}_{i,nom}(t) = c_1c_2c_3s_i(t) - (c_1c_2 + c_1c_3 + c_2c_3)v_i(t) + \frac{c}{\tau}T_i(t) - (c_1 + c_2 + c_3)\left(cT_i(t) - g - dv_i(t)^2\right) + 2dv_i(t) \times \left(cT_i(t) - g - dv_i(t)^2\right) + c_1c_2v_{i-1}(t), \tag{5}$$

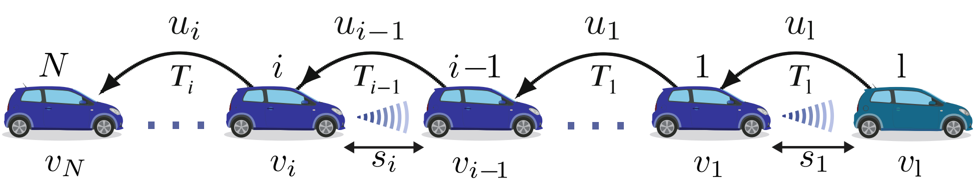

FIGURE 1 Homogeneous platoon of N vehicles, with dynamics described by (1)–(3), following a leader. Each vehicle measures its own speed, the relative speed with the preceding vehicle, and the spacing with respect to the preceding vehicle. Information about the control input and engine torque of each vehicle is transmitted only to the following vehicle, via V2V communication.

where c_1, c_2, c_3 are positive design parameters, which satisfy

$$c_2 = \frac{c_1}{c_1 h - 1}, \tag{6}$$

with $c_1 > \frac{1}{h}$, where $h > 0$ is the desired time-headway. The construction of feedback law (5) relies on the design procedure developed by Krstic and Bement[24] for general, strict-feedback nonlinear systems. In more detail, the construction relies on introduction of transformations (A2)–(A4), which transform the closed-loop system into (A5)–(A7). This procedure results in a proper modification of the design in Krstic and Bement,[24] to making it suitable as a CTH ACC design.

In fact, such a design in the nominal, delay-free case, may guarantee positivity of speed and spacing states, individual vehicle stability, string stability, and tracking of the desired speed/spacing (this could be seen specializing the result stated in Theorem 1 to the case $D = 0$). In more detail, these attributes are enabled by the design in Krstic and Bement,[24] through incorporating in the feedback law the speed of the preceding vehicle v_{i-1} and choosing the gain c_2 as in (6), as well as choosing s_i as an output that we require to remain positive only, without regulating it at the safety region (where $s_i > 0$) boundary. In particular, the specific choice (6) is made in order to guarantee tracking of the desired spacing that corresponds to a given time-headway, that is, to guarantee that the equilibrium spacing corresponding to a constant, equilibrium speed v^* is $s^* = hv^*$. In fact, with the choice (6) (that implies $c_1 + c_2 = hc_1c_2$) the feedback laws (5) are of CTH type, since they can be written as

$$\begin{aligned}\bar{u}_{i,nom}(t) &= c_1c_2c_3(s_i(t) - hv_i(t)) + c_1c_2(v_{i-1}(t) - v_i(t)) - (c_1 + c_2 + c_3)\left(cT_i(t) - g - dv_i(t)^2\right) + 2dv_i(t) \\ &\quad \times \left(cT_i(t) - g - dv_i(t)^2\right) + \frac{c}{\tau}T_i(t). \end{aligned} \tag{7}$$

2.3 | Predictor-feedback CACC design

One of the key design ideas of a predictor-feedback law is to construct an implementable formula for the future state of the system considered; in the present case, for the future state of vehicles. This can be achieved utilizing a model for the dynamics of each vehicle, together with measurements of its own control input history (over a horizon of D time units), as well as of the history of the control input of its preceding vehicle. The predictor states are then employed in a nominal, delay-free design. In the present case, this results in a control law of the form $u_i = f\left(q_{i,1}, q_{i,2}, q_{i,3}, q_{i,4}\right)$, where $q_i = \begin{bmatrix} q_{i,1} & q_{i,2} & q_{i,3} & q_{i,4}\end{bmatrix}^{\mathrm{T}}$ are the D-time units ahead predictor states of $\bar{x}_i = \begin{bmatrix} s_i & v_i & v_{i-1} & T_i\end{bmatrix}^{\mathrm{T}}$. Further details on construction of predictor states for general nonlinear systems can be found, for example, in the book of Bekiaris-Liberis and Krstic.[28]

Employing the nominal design (4), (5), the predictor-feedback laws for system (1)–(3) are thus given by (4), (5) with

$$u_i(t) = \frac{\tau}{c}\bar{u}_i(t) \tag{8}$$

$$\begin{aligned}\bar{u}_i(t) &= c_1c_2c_3q_{i,1}(t) - (c_1c_2 + c_1c_3 + c_2c_3)q_{i,2}(t) + \frac{c}{\tau}q_{i,4}(t) - (c_1 + c_2 + c_3)\left(cq_{i,4}(t) - g - dq_{i,2}(t)^2\right) + 2dq_{i,2}(t) \\ &\quad \times \left(cq_{i,4}(t) - g - dq_{i,2}(t)^2\right) + c_1c_2q_{i,3}(t). \end{aligned} \tag{9}$$

The formulae of the predictor states are given, for $i = 1, \dots, N$, by

$$q_{i,1}(t) = s_i(t) + \int_{t-D}^{t}\left(q_{i,3}(s) - q_{i,2}(s)\right)ds, \tag{10}$$

$$q_{i,2}(t) = v_i(t) + \int_{t-D}^{t}\left(-dq_{i,2}(s)^2 - g + cq_{i,4}(s)\right)ds \tag{11}$$

$$q_{i,4}(t) = \mathrm{e}^{-\frac{D}{\tau}}T_i(t) + \frac{1}{\tau}\int_{t-D}^{t}\mathrm{e}^{-\frac{t-\theta}{\tau}}u_i(\theta)d\theta, \tag{12}$$

and

$$q_{i,3}(t) = v_{i-1}(t) + \int_{t-D}^{t}\left(-dq_{i,3}(s)^2 - g + cq_{i-1,4}(s)\right)ds, \quad i = 2, \dots, N \tag{13}$$

$$q_{1,3}(t) = v_1(t) + \int_{t-D}^{t} q_{1,4}(s)ds \tag{14}$$

$$q_{1,4}(t) = \mathrm{e}^{-\frac{D}{\tau}} T_1(t) + \frac{1}{\tau}\int_{t-D}^{t} \mathrm{e}^{-\frac{t-\theta}{\tau}} u_1(\theta)d\theta, \tag{15}$$

with $q_{i,1}$, $q_{i,2}$, and $q_{i,4}$, being initialized for $\theta \in [-D, 0)$ as

$$q_{i,1}(\theta) = s_i(0) + \int_{-D}^{\theta} \left(q_{i,3}(s) - q_{i,2}(s)\right)ds, \tag{16}$$

$$q_{i,2}(\theta) = v_i(0) + \int_{-D}^{\theta} \left(-dq_{i,2}(s)^2 - g + cq_{i,4}(s)\right)ds \tag{17}$$

$$q_{i,4}(\theta) = \mathrm{e}^{-\frac{\theta+D}{\tau}} T_i(0) + \frac{1}{\tau}\int_{-D}^{\theta} \mathrm{e}^{-\frac{\theta-s}{\tau}} u_{i_0}(s)ds, \tag{18}$$

for $i = 1, \dots, N$ and $q_{i,3}$, $q_{1,4}$ being initialized as

$$q_{i,3}(\theta) = v_{i-1}(0) + \int_{-D}^{\theta} \left(-dq_{i,3}(s)^2 - g + cq_{i-1,4}(s)\right)ds, \quad i = 2, \dots, N \tag{19}$$

$$q_{1,3}(\theta) = v_1(0) + \int_{-D}^{\theta} q_{1,4}(s)ds \tag{20}$$

$$q_{1,4}(\theta) = \mathrm{e}^{-\frac{\theta+D}{\tau}} T_1(0) + \frac{1}{\tau}\int_{-D}^{\theta} \mathrm{e}^{-\frac{\theta-s}{\tau}} u_{1_0}(s)ds. \tag{21}$$

The predictor states (10)–(15) involved in control design (8) can be numerically computed via a numerical approximation scheme.[29] Note that the nominal, delay-free control design (4) is of ACC type. However, for constructing the predictor state for the preceding vehicle's speed, measurements of the control input u_{i-1} and torque T_{i-1} of the preceding vehicle have to be available. This is possible through V2V communication. The rest of the measurements required for implementation of (8), that is, s_i, v_i, u_i, v_{i-1}, and T_i are obtained from on-board sensors. Note also that the controller of the ego vehicle i computes (10)–(15), which, in fact, incorporates computation of five in total predictor states, as it also involves computation of $q_{i-1,4}$ (according to relations (12), (15)).

3 | POSITIVITY OF SPEED/SPACING STATES AND STRING STABILITY UNDER NONLINEAR PREDICTOR-FEEDBACK CACC

3.1 | String stability definition

Definition 1. An interconnected system of vehicles, indexed by $i = 1, \dots, N$, following each other in single lane without passing, with dynamics described by (1)–(3), is $\mathcal{L}_\infty$ string stable if the following hold for $i = 1, \dots, N$ (in the delay-free case, there also exist similar and more general definitions[30–33])

$$\|\tilde{v}_i\|_\infty \leq \gamma_0(\|\tilde{v}_{i-1}\|_\infty) + \gamma_1\left(\left|\tilde{s}_{i_0}\right|\right) + \gamma_2\left(\left|\tilde{v}_{i_0}\right| + \sqrt{\frac{c}{d}}\sqrt{\left|\tilde{T}_{i,0}\right| + \sup_{\theta\in[-D,0]}\left|\tilde{u}_{i_0}(\theta)\right|}\right) + \gamma_3\Bigg(\left|\tilde{v}_{i-1_0}\right| + \sqrt{\frac{c}{d}}\sqrt{\left|\tilde{T}_{i-1,0}\right| + \sup_{\theta\in[-D,0]}\left|\tilde{u}_{i-1_0}(\theta)\right|}\Bigg), \tag{22}$$

where $\tilde{v}_i = v_i - v^*$, $\tilde{v}_1 = v_1 - v^*$, $\tilde{T}_1 = T_1$, $\tilde{T}_i = T_i - u^*$, $\tilde{s}_i = s_i - s^*$, $\tilde{u}_i = u_i - u^*$, $\tilde{u}_1 = u_1$, $\|v_i - v^*\|_\infty = \sup_{t\geq 0}|v_i(t) - v^*|$, $s^* = hv^*$, $u^* = \frac{g+v^{*2}}{c}$, and $\gamma_0 : [0, +\infty) \to [0, +\infty)$, with $\gamma_0(r) \leq r$, $\gamma_1 : [0, +\infty) \to [0, +\infty)$, and $\gamma_2, \gamma_3 : \left[0, \frac{1}{dD}\right) \to [0, +\infty)$ are class $\mathcal{K}$ functions.[34]

Other definitions are also possible, in particular, which may involve studying disturbance propagation (upstream in the platoon) of spacing errors or accelerations. For simplicity we provide definition of string stability with respect to speed errors only and since this is the most commonly employed definition[†].

In string stability definition (22), the last term appears due to the presence of input delay (this becomes clear within the proof of Theorem 1 in Appendix A; see relations (A38), (A39)). This is explained by the fact that, during the dead-time interval, according to (1)–(3) the speed dynamics of the ego vehicle depend on the initial conditions of its own speed, torque, and actuator state. Whereas the respective spacing dynamics are also affected by the preceding vehicle's speed within an interval of D time units, which, in turn, depends on the initial conditions of the preceding vehicle's speed, torque, and respective actuator state. This gives rise to the last term in (22), which would not appear in the string stability definition if $D = 0$. In fact, string stability should be viewed more as a property related to the platoon only during closed-loop operation (i.e., for $t \geq D$) of each individual vehicular system (i.e., also consistent with the delay-free case). This is attributed to the fact that, during the dead-time interval, each individual vehicle operates in open loop, and thus, its speed dynamics are affected only by initial conditions.

3.2 | Definition of comparison functions of initial conditions

In order to state the main result of the paper, which is presented in the next subsection, we need to define certain functionals of the initial conditions. These explicitly given functionals are utilized in order to explicitly quantify the range of allowable initial conditions (and control gains) that guarantee positivity of speed and spacing states both during open- and closed-loop operation. We define the following functionals for $i = 1, \ldots, N$

$$\delta_i\left(T_{i_0}, u_{i_0}\right) = \max\{0, g - c\bar{\delta}_i\left(T_{i_0}, u_{i_0}\right)\} \tag{23}$$

$$\zeta_i\left(T_{i_0}, u_{i_0}\right) = \max\{0, c\bar{\zeta}_i\left(T_{i_0}, u_{i_0}\right) - g\} \tag{24}$$

$$\bar{\delta}_i\left(T_{i_0}, u_{i_0}\right) = \min\left\{T_{i_0}, T_{i_0}\mathrm{e}^{-\frac{D}{\tau}}\right\} + \left(1 - \mathrm{e}^{-\frac{D}{\tau}}\right)\min\left\{\inf_{s\in[-D,0]} u_{i_0}(s), 0\right\} \tag{25}$$

$$\bar{\delta}_l\left(T_{l_0}, u_{l_0}\right) = \min\left\{T_{l_0}, T_{l_0}\mathrm{e}^{-\frac{D}{\tau}}\right\} + \left(1 - \mathrm{e}^{-\frac{D}{\tau}}\right)\min\left\{\inf_{s\in[-D,0]} u_{l_0}(s), 0\right\} \tag{26}$$

$$\bar{\zeta}_i\left(T_{i_0}, u_{i_0}\right) = \max\left\{T_{i_0}, T_{i_0}\mathrm{e}^{-\frac{D}{\tau}}\right\} + \left(1 - \mathrm{e}^{-\frac{D}{\tau}}\right)\max\left\{\sup_{s\in[-D,0]} u_{i_0}(s), 0\right\} \tag{27}$$

$$m_i\left(v_{i_0}, T_{i_0}, u_{i_0}\right) = -\frac{\sqrt{\delta_i}}{\sqrt{d}}\tan\left(\sqrt{d\delta_i}D - \tan^{-1}\left(v_{i_0}\frac{\sqrt{d}}{\sqrt{\delta_i}}\right)\right) \tag{28}$$

$$m_0\left(v_{l_0}, T_{l_0}, u_{l_0}\right) = v_{l_0} + D\min\left\{0, \bar{\delta}_l\left(T_{l_0}, u_{l_0}\right)\right\} \tag{29}$$

$$M_i\left(v_{i_0}, T_{i_0}, u_{i_0}\right) = \max\left\{v_{i_0}, \frac{\sqrt{\zeta_i}}{\sqrt{d}}\frac{v_{i_0} + \frac{\sqrt{\zeta_i}}{\sqrt{d}} + \left(v_{i_0} - \frac{\sqrt{\zeta_i}}{\sqrt{d}}\right)\mathrm{e}^{-2\sqrt{\zeta_i d}D}}{v_{i_0} + \frac{\sqrt{\zeta_i}}{\sqrt{d}} - \left(v_{i_0} - \frac{\sqrt{\zeta_i}}{\sqrt{d}}\right)\mathrm{e}^{-2\sqrt{\zeta_i d}D}}\right\}. \tag{30}$$

3.3 | Statement of main result

Theorem 1. *Consider a platoon of vehicles with dynamics modeled by (1)–(3), under the control laws (8) with (9)–(21). Assume that the leading vehicle satisfies $m_0 > 0$, $v_l(t) > 0$, for all $t \geq 0$, $v_l \in \mathcal{L}_\infty$, and $u_l \in C[-D, +\infty)$, where m_0 is defined in (29). Then for any $D \geq 0$, $h > 0$, with the choice of control gains such that $c_1 > \frac{1}{h}$, c_2 satisfying (6), and $c_3 > 0$, the closed-loop systems' solutions satisfy for $i = 1, 2, \ldots, N$*

$$s_i(t) > 0, \quad v_i(t) > 0, \quad \text{for all } t \geq 0, \tag{31}$$

provided that the initial conditions $s_{i_0}, v_{i_0} \in \mathbb{R}_+$, $T_{i_0} \in \mathbb{R}$, *and* $u_{i_0} \in C[-D, 0]$[‡], $i = 1, 2, \dots, N$, *satisfy*

$$v_{i_0} > \tan\left(\sqrt{d\delta_i} D\right) \frac{\sqrt{\delta_i}}{\sqrt{d}} \tag{32}$$

$$s_{i_0} > \frac{M_i}{c_1} - D \min\{0, m_{i-1} - M_i\} \tag{33}$$

$$c_1 m_i > -c\overline{\delta}_i + g + dM_i^2 \tag{34}$$

$$c\overline{\zeta}_i < -\frac{c_1^2 h}{c_1 h - 1} M_i + g + dm_i^2 + \frac{c_1^2}{c_1 h - 1}\left(s_{i_0} + D \min\{0, m_{i-1} - M_i\}\right) \tag{35}$$

$$\delta_i < \frac{\pi^2}{4D^2 d}, \tag{36}$$

where $\delta_i, \zeta_i, \overline{\delta}_i, \overline{\zeta}_i, m_i$, *and* M_i *are defined in* (23)–(30). *Furthermore, if, in addition, the initial conditions satisfy*

$$\left|\tilde{v}_{i_0}\right| + \sqrt{\frac{c}{d}} \sqrt{\left|\tilde{T}_{i_0}\right| + \sup_{\theta \in [-D, 0]} \left|\tilde{u}_{i_0}(\theta)\right|} < \frac{1}{Dd}, \tag{37}$$

for $i = 0, \dots, N$[§], *then the platoon is* $\mathcal{L}_\infty$ *string stable and, for constant leader's speed* v^*, *each individual vehicular system is asymptotically stable and zero steady-state tracking errors are achieved.*

Proof. The proof can be found in Appendix A. ∎

It should be noted that we are not aware of an ACC design that capitalizes on the general design procedure developed by Krstic and Bement[24] even when $D = 0$. Because we employ predictor feedback (and thus, for $t \geq D$ the delay is completely compensated), in combination with the procedure of Krstic and Bement,[24] the closed-loop transformed system is linear. In particular, it is such that the respective input-to-output map is linear, where v_{i-1} acts as input and v_i as output (this is the system consisting of (A7), (A11), and (A12)). Thus, one could (as long as the initial conditions are at equilibrium) study also $\mathcal{L}_2$ string stability as well as string stability with respect to spacing errors employing the corresponding transfer function. This transfer function is given as $G(s) = \frac{c_1 c_2}{(s+c_1)(s+c_2)}$, which satisfies $G(0) = 1$ and has two negative, real poles, and thus, it corresponds to a non-negative impulse response with $\sup_\omega |G(j\omega)| \leq 1$.

It should be noted that the nonlinear, predictor-feedback CACC design methodology presented is flexible and one could modify it to incorporate different, nominal (for the delay-free case) ACC designs and models for vehicles' dynamics. For example, predictor feedback could be combined with nominal ACC laws constructed utilizing Control Barrier Functions,[35] aiming, for example, at satisfying the safety condition $s_i \geq hv_i$ (instead of $s_i > 0$ considered here), and with other, nonlinear ACC designs.[19,21,22,30,31] Note that in the former case, stabilization at the equilibrium $s_i = hv_i$ (i.e., at the boundary of the safety region) is achieved from initial conditions such that $s_{i_0} \geq hv_{i_0}$. While in the present case, because in (33) $\frac{1}{c_1} < h$ and since (35) could possibly be satisfied with $s_{i_0} < hv_{i_0}$, initial conditions such that $s_{i_0} < hv_{i_0}$ could be considered. Nevertheless, this is dependent also on the initial conditions T_{i_0} and u_{i_0} according to (35).

Predictor feedback could be also combined with other nonlinear models for vehicles' dynamics, such as, for example, with the model considered in the work of Ioannou and Chien.[19] In such a case, one could, for example, modify the CACC law developed by Bekiaris-Liberis[11] to achieve, during closed-loop operation, linear closed-loop dynamics, adding a feedback linearizing component in the original CACC law. This is possible because the nonlinearities in the model in the work of Ioannou and Chien[19] are matched with the control input. This would enable to subsequently employ the results developed by Bekiaris-Liberis[11] to study regulation, stability, string stability, and positivity of speed and spacing states. The challenge would be to derive explicit estimates on open-loop solutions for the model in the work of Ioannou and Chien,[19] to guarantee positivity of speed/spacing states during open-loop operation and that the finite escape time phenomenon is avoided.

3.4 Discussion on conditions (32)–(37) of theorem 1

Requirements (32)–(37) on initial conditions may be, in practice, restrictive. Nevertheless, they are necessary to theoretically establish positivity and boundedness of speed and spacing states, in view of the nonlinear vehicle model (1)–(3) and the presence of input delay considered. In particular, since within open-loop operation, during the dead-time interval, each individual, feedback control input does not affect the respective vehicular system, one should necessarily impose conditions on initial conditions to guarantee both positivity of speed/spacing states and that there is no finite-escape time phenomenon arising (conditions (32), (33), and (36)). The latter may be viewed more as a theoretical property, in view of the practical aspect of the vehicular systems considered. To guarantee positivity of speed and spacing states during closed-loop operation the initial conditions should satisfy restrictions (33)–(35). In particular, conditions (33) and (35) could be satisfied for a sufficiently large s_{i_0}; while (34) could be satisfied further restricting the initial conditions v_{i_0}, T_{i_0}, and u_{i_0}. In fact, in practice, conditions (33) and (34) could be also satisfied with a sufficiently large choice for the control gain c_1[¶], possibly depending on initial conditions[#]. Condition (37) (that can be satisfied restricting the initial conditions) arises when deriving estimates on open-loop solutions, with respect to their deviation from the desired equilibrium, and thus, it may be viewed as a condition for stabilization and string stability. We further illustrate in simulation both the theoretical guarantees derived and the imposed conditions, as well as the practical significance of the CACC design.

Even though conditions (32)–(37) may appear as more of theoretical nature, they are also reasonable from a practical viewpoint. For example, condition (33) implies that the initial condition for spacing should be sufficiently large, depending on the respective initial condition for speed. This is a reasonable requirement from the viewpoint of collision avoidance; see also, for instance, the work of Molnar et al.[35] While condition (34) implies that the initial speed (or the control gain c_1) should be sufficiently large, depending on the maximum possible deceleration (during the dead-time interval). This is a reasonable requirement from the viewpoint of guaranteeing non-negativity of speed in practice, despite potentially large decelerations.

4 SIMULATION RESULTS

We illustrate here the safety, stability, and string stability properties of the nonlinear, predictor-feedback CACC design with a platoon of ten vehicles. We consider a scenario in which $D = 0.5$, $h = 0.75$, while for the vehicles we set $d = 0.00025$, $c = 0.0005$, $g = 0.002$, and $\tau = 0.3$, which are realistic values for vehicles.[10,13,19] We choose $c_1 = 3$, $c_2 = \frac{c_1}{c_1 h - 1}$, and $c_3 = 0.5$. We consider a scenario in which a vehicle cuts in the platoon, which is demonstrated assuming initial conditions for the torques and control inputs at equilibrium, namely, $T_{i_0} = \frac{dv_{i_0}^2 + g}{c}$, $i = 1, \dots, 9$, $T_{1_0} = 0$, $u_{i_0}(s) = T_{i_0}$, for $s \in [-D, 0)$ and $i = 0, \dots, 9$; whereas for the speeds we set $v_{i_0} = 15$, $i = 1, \dots, 9$, $v_{1_0} = \frac{2}{3} v_{1_0}$ and for the spacings we set $s_{i_0} = hv_{i_0}$, $i = 2, \dots, 9$, and $s_{1_0} = \frac{4}{5} hv_{1_0}$. We further consider that the leading vehicle performs a deceleration/acceleration maneuver. Thus, this scenario illustrates the effectiveness of the proposed design with respect to both initial conditions deviations from equilibrium and leading vehicle's maneuvers. As it is shown in Figure 2, positivity of speed and spacing states is achieved, while the responses to the leading vehicle's maneuvers feature no overshoot, as result of the achieved $\mathcal{L}_\infty$ string stability. Furthermore, regulation of speed and spacing states at the desired, reference values is also achieved, as a result of the achieved asymptotic stability.

For the same scenario we also perform simulations in the case in which the delay value is uncertain. In Figure 3 we show the responses of the vehicles when the actual actuator delay is $D_r = 0.6$, whereas the delay value available to the designers (and employed in the predictor-feedback CACC laws) is $D = 0.5$. One can observe that performance is still satisfactory, despite, in general, the responses being more oscillatory. This is consistent with the delay-robustness properties of general, nonlinear predictor feedbacks[36] and of predictor-feedback CACC designs,[11] which guarantee that stability and regulation are still achieved. Note that in the work of Bekiaris-Liberis[11] it is shown that string stability in $\mathcal{L}_2$, under uncertainty in the value of actuators delays, is preserved; nevertheless, this result concerns linear vehicle dynamics. In the present case, it is not straightforward to obtain a respective result because both the vehicles' model and CACC design are nonlinear. We also note that, in the case in which the actual input delay is $D_r = 0.5$, whereas the designer presumes that there is no delay, that is, $D = 0$, thus, essentially, the nominal, delay-free ACC law is employed without predictor feedback, stabilization is not possible leading to unrealistic vehicles' responses.

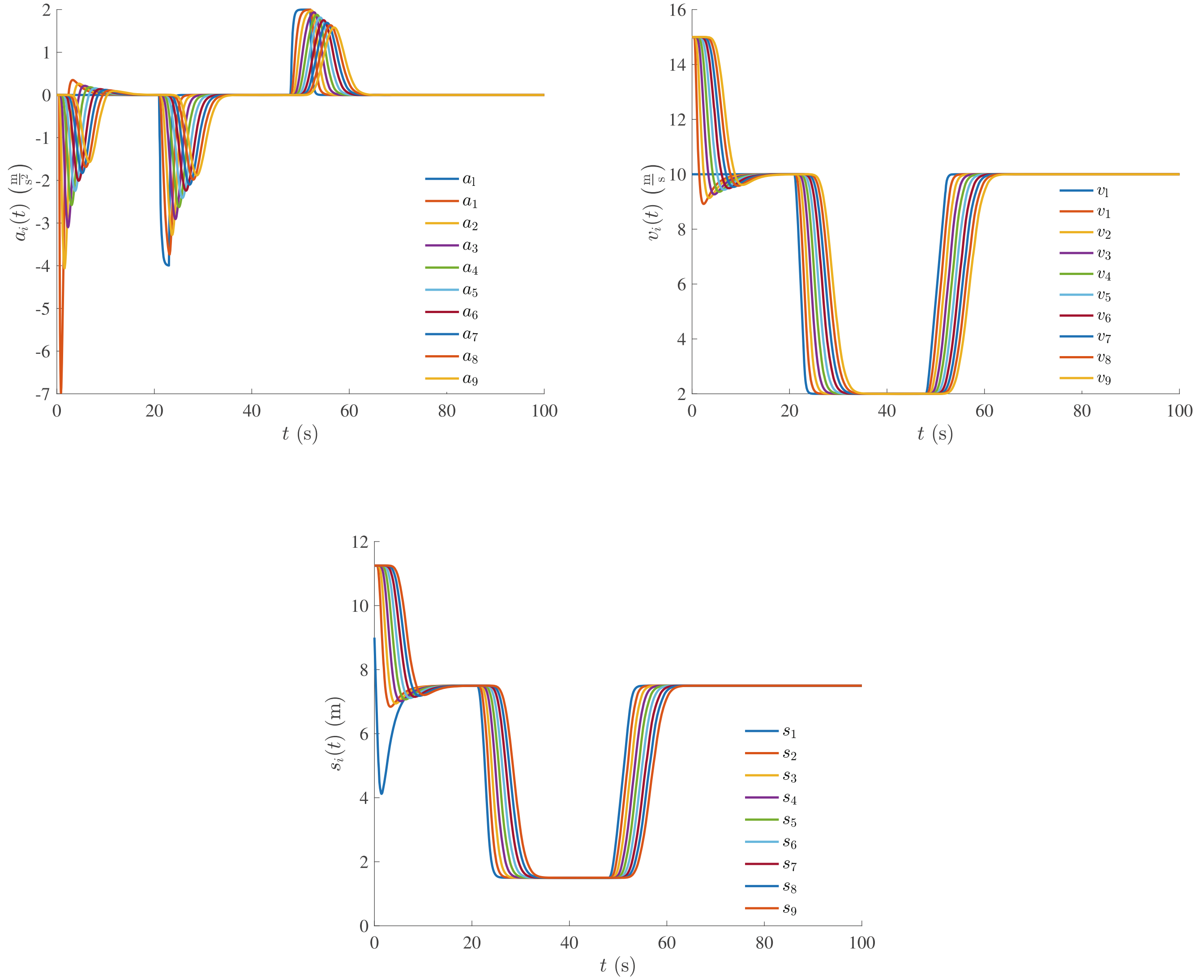

FIGURE 2 Acceleration (top-left), speed (top-right), and spacing (bottom) of nine vehicles, with dynamics described by (1)–(3), following a leader that cuts in the platoon and, subsequently, it performs an acceleration/deceleration maneuver, under the nonlinear predictor-feedback CACC laws (8). Accelerations are given by the right-hand side of (2) for $i = 1, \ldots, N$ and by the right-hand side of equation $\dot{v}_l = T_l$, where $\dot{T}_l(t) = -\frac{1}{\tau}T_l(t) + \frac{1}{\tau}u_l(t-D)$, for the leading vehicle.

5 | CONCLUSIONS AND DISCUSSION

We presented a nonlinear predictor-feedback CACC design for homogeneous vehicular platoons in which each individual vehicular system is described by a third-order, nonlinear system with input delay. The delay-compensating property of the design results in positivity of speed and spacing states, as well as asymptotic stability of each individual vehicular system and string stability of the platoon. All of which are important requirements for safe and efficient operation of vehicular platoons. The guarantees are proved deriving explicit estimates on solutions and utilizing the delay-compensating property of predictor feedback as well as the safety properties of the underlying, nominal (for the delay-free case) design.

To avoid burying the key contribution of the paper, which is delay compensation for vehicular platoons via nonlinear predictor-feedback CACC, in technical details, we consider homogeneous platoons, which have been successfully used for ACC/CACC design and analysis under delay effects in existing works.[2,6,9,10,15,16,17] There is no conceptual obstacle to extend the results presented to the case of non-homogeneous platoons, as long as the actuator delay is identical in all vehicles (and the parameters d, g, τ, c of the preceding vehicle are communicated to the ego vehicle). Addressing the case in which the input delay in each individual vehicular system is different, it is a quite different problem. The reason is that it requires development of a new predictor-feedback CACC design, which employs (exact) predictor states over different

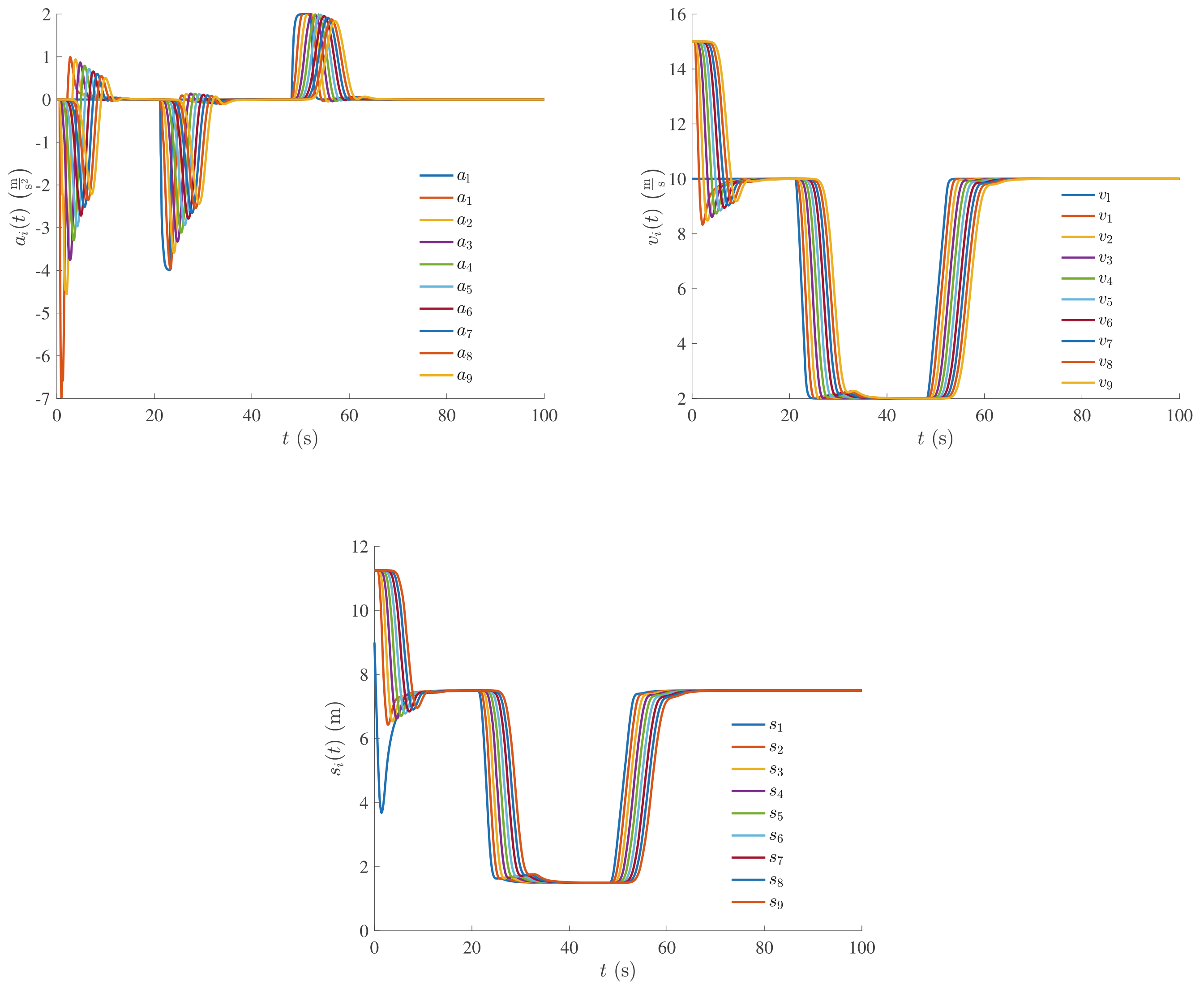

FIGURE 3 Acceleration (top-left), speed (top-right), and spacing (bottom) of nine vehicles, with dynamics described by (1)–(3) and a +20% uncertainty in the delay value available to the designers, following a leader that cuts in the platoon and, subsequently, it performs an acceleration/deceleration maneuver, under the nonlinear predictor-feedback CACC laws (8). Accelerations are given by the right-hand side of (2) for $i = 1, \ldots, N$ and by the right-hand side of equation $\dot{v}_l = T_l$, where $\dot{T}_l(t) = -\frac{1}{\tau}T_l(t) + \frac{1}{\tau}u_l(t-D)$, for the leading vehicle.

prediction horizons (corresponding to different delay values). Thus, it cannot be obtained in a straightforward manner from the design presented here. Such a design would be more complex and would, potentially, require additional V2V communication capabilities. In particular, it would require communication with more than one vehicle ahead, because for constructing the predictors for the states of the preceding vehicle, not only its model but also its CACC law would be needed. The starting point for the construction of such a CACC law may be the results developed by Bekiaris-Liberis and Krstic,[37] which provide the predictor-feedback design for general, multi-input nonlinear systems with distinct input delays. As an alternative, one could still apply the CACC design developed here, employing a single delay value, to the case of distinct input delays. Since nonlinear predictor feedbacks are robust to delay uncertainties,[36] it is expected that such a design may still exhibit a satisfactory performance.

In the present paper we do not address communication delay that may appear due to V2V communication. The reason is that, in such a case, it is not clear how to construct the predictor state for the speed of the preceding vehicle, which would require transmission of the current control input history and acceleration information, from the preceding vehicle to the ego vehicle. The latter is not possible under communication delay. However, in the work of Samii and Bekiaris-Liberis[38] we establish that, for the linear case under input delay, asymptotic stability and string stability of predictor-feedback CACC are robust to the presence of communication delay. Simultaneous compensation of input and communication delays, in platoons with vehicles featuring nonlinear dynamics, is a theoretically and practically significant problem, which we are currently investigating.

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CONFLICT OF INTEREST STATEMENT

The author declares no potential conflict of interests.

DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

ENDNOTES

*Here we review only papers dealing with delay compensation by design and not with studying robustness to small delay values.

†However, string stability with respect to spacing errors and $\mathcal{L}_2$ string stability can be also established in a straightforward manner as byproduct of the predictor-feedback CACC design developed (see the discussion in the paragraph immediately after the statement of Theorem 1 in Section 3.3).

‡In fact, $u_{i_0} \in C[-D, 0]$ being compatible with the feedback laws (8).

§The fact that condition (37) also includes the leading vehicle is made only for consistency, between the string stability definition and the respective proof, and it could be removed. However, if we do not include $i = 0$, then definition (22) should be modified such that γ_3 is different for $i = 1$. Since the paper may already appear as being technically cumbersome and given that it addresses a specific application, we do not modify the string stability definition to reduce technical burden, sacrificing some generality degree.

¶This is not true for (35) because one has a degree of freedom less, for satisfying (35), because the gain c_2 is chosen as function of c_1, according to (6).

#Even though, theoretically, in such a case, γ_1, γ_2, and γ_3 in (22) may not be of class $\mathcal{K}$ anymore, but rather only nonnegative-valued, continuous functionals of initial conditions that are zero at equilibrium; while the convergence rate to the desired equilibrium may not be uniform, with respect to initial conditions, anymore.

‖In the subsequent analysis, for simplicity of presentation of the explicit solutions and their estimates derived, we consider the case in which relation $c_1 \neq c_2 \neq c_3$ holds. This is without loss of generality and the analysis still holds (with simple modifications) in the case in which two, among the control gains, may be chosen as equal to each other.

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APPENDIX A. PROOF OF THEOREM 1

A.1 Proof of positivity of speed and spacing states

The signals $q_i = \begin{bmatrix} q_{i,1} & q_{i,2} & q_{i,3} & q_{i,4} \end{bmatrix}^{\mathrm{T}}$ in (10)–(15), initialized according to (16)–(21), satisfy $q_i(t) = \overline{x}_i(t + D)$, with $\overline{x}_i = \begin{bmatrix} s_i & v_i & v_{i-1} & T_i \end{bmatrix}^{\mathrm{T}}$, for all $t \geq 0$.[28] Therefore, under the feedback laws (8), (9) for $t \geq D$ the closed-loop systems satisfy (1), (2), and

$$\dot{T}_i(t) = -\frac{1}{\tau} T_i(t) + \frac{1}{\tau} u_{i,nom}(t), \quad i = 1, \ldots, N, \tag{A1}$$

where $u_{i,nom}$ is defined in (4), (5). Defining for $i = 1, \dots, N$ the transformations (see the work of Krstic and Bement[24])

$$z_{i,1} = -s_i \tag{A2}$$
$$z_{i,2} = v_i - c_1 s_i \tag{A3}$$
$$z_{i,3} = cT_i + (c_1 + c_2)v_i - g - dv_i^2 - c_1 c_2 s_i, \tag{A4}$$

we get that for $t \geq D$ the following hold

$$\dot{z}_{i,1}(t) = -c_1 z_{i,1}(t) + z_{i,2}(t) - v_{i-1}(t) \tag{A5}$$
$$\dot{z}_{i,2}(t) = -c_2 z_{i,2}(t) + z_{i,3}(t) - c_1 v_{i-1}(t) \tag{A6}$$
$$\dot{z}_{i,3}(t) = -c_3 z_{i,3}(t), \tag{A7}$$

for $i = 1, \dots, N$. In order to guarantee positivity of spacing and speed states for $t \geq D$ it should hold that $z_{i,1}(t) < 0$ and $z_{i,2}(t) - c_1 z_{i,1}(t) > 0$, respectively, for all $t \geq D$ and $i = 1, \dots, N$. By induction and under assumption $v_l(t) > 0$, for all $t \geq 0$, the form of the closed-loop, transformed systems (A5)–(A7) guarantees that $z_{i,j}(t) < 0$, for all $t \geq D$, $j = 1, 2, 3$, and $i = 1, \dots, N$, hold provided that

$$z_{i,j}(D) < 0, \quad j = 1, 2, 3, \quad i = 1, \dots, N. \tag{A8}$$

Furthermore, relations $v_i(t) = z_{i,2}(t) - c_1 z_{i,1}(t) > 0$, $i = 1, \dots, N$, hold for all $t \geq D$ provided that

$$z_{i,3}(D) > c_2 z_{i,2}(D), \quad i = 1, \dots, N \tag{A9}$$
$$z_{i,2}(D) > c_1 z_{i,1}(D), \quad i = 1, \dots, N. \tag{A10}$$

This can be seen as follows. The speed states $v_i = z_{i,2} - c_1 z_{i,1}$, $i = 1, \dots, N$, satisfy

$$\dot{v}_i(t) = -c_1 v_i(t) + \bar{g}_i(t), \tag{A11}$$

where $\bar{g}_i = z_{i,3} - c_2 z_{i,2}$, which follows from (A5), (A6). Hence, the speed states v_i remain positive for $t \geq D$ provided that (A10) and $z_{i,3}(t) - c_2 z_{i,2}(t) > 0$, $t \geq D$, hold. The latter holds by induction in view of (A9) and the fact that $z_{i,3}(t) < 0$, $t \geq D$ (in view of (A7), (A8)), because the signals $\bar{g}_i(t) = z_{i,3}(t) - c_2 z_{i,2}(t)$, $i = 1, \dots, N$, satisfy

$$\dot{\bar{g}}_i(t) = -c_2 \bar{g}_i(t) - c_3 z_{i,3}(t) + c_1 c_2 v_{i-1}(t). \tag{A12}$$

Therefore, it remains to establish (A8)–(A10), which can be written using (6) and (A2)–(A4) as

$$s_i(D) > 0 \tag{A13}$$
$$v_i(D) > 0 \tag{A14}$$
$$c_1 s_i(D) > v_i(D) \tag{A15}$$
$$c_1 v_i(D) > -cT_i(D) + g + dv_i(D)^2 \tag{A16}$$
$$cT_i(D) < -\frac{c_1^2 h}{c_1 h - 1} v_i(D) + g + dv_i(D)^2 + \frac{c_1^2}{c_1 h - 1} s_i(D), \tag{A17}$$

for $i = 1, \dots, N$. To satisfy (A13)–(A17) one should restrict the initial conditions, although (A15), (A16) could be also satisfied (under (A13), (A14)) with a sufficiently large choice for the control gain c_1. For either case, we proceed next in establishing (A13)–(A17) together with positivity of speed and spacing states during the dead-time interval. During the dead-time interval, from (3) we get for $i = 1, \dots, N$ that

$$T_i(t) = e^{-\frac{t}{\tau}} T_{i_0} + \frac{1}{\tau} \int_0^t e^{-\frac{t-s}{\tau}} u_{i_0}(s - D) ds. \tag{A18}$$

Hence, it holds that $\bar{\delta}_i \leq T_i(t) \leq \bar{\zeta}_i$, $t \in [0, D]$, where $\bar{\delta}_i$ and $\bar{\zeta}_i$ are defined in (25) and (27), respectively. With definitions (23), (24) and defining $v_i = -y_i$, we get from (2) that

$$\dot{y}_i(t) \leq dy_i(t)^2 + \delta_i, \quad i = 1, \ldots, N. \tag{A19}$$

Thus, under assumptions (32) and (36), using the comparison principle (see, e.g., Lemma 3.4 in the book of Khalil[34]) we obtain

$$v_i(t) \geq -\frac{\sqrt{\delta_i}}{\sqrt{d}} \tan\left(\sqrt{d\delta_i}t - \tan^{-1}\left(v_{i_0}\frac{\sqrt{d}}{\sqrt{\delta_i}}\right)\right). \tag{A20}$$

Note that under assumptions (32) and (36) we obtain that $v_i(t) > 0$, for all $0 \leq t \leq D$, because $-\frac{\pi}{2} < \sqrt{d\delta_i}t - \tan^{-1}\left(v_{i_0}\frac{\sqrt{d}}{\sqrt{\delta_i}}\right) < 0$, for all $0 \leq t \leq D$, and hence relation (A14) is established. Similarly, with definition (24) we get from (2) that $\dot{v}_i(t) \leq -dv_i(t)^2 + \zeta_i$, $i = 1, \ldots, N$, and hence, from the comparison principle we get for all $t \in [0, D]$ that

$$v_i(t) \leq \frac{\sqrt{\zeta_i}}{\sqrt{d}} \frac{v_{i_0} + \frac{\sqrt{\zeta_i}}{\sqrt{d}} + \left(v_{i_0} - \frac{\sqrt{\zeta_i}}{\sqrt{d}}\right)e^{-2\sqrt{\zeta_i d}t}}{v_{i_0} + \frac{\sqrt{\zeta_i}}{\sqrt{d}} - \left(v_{i_0} - \frac{\sqrt{\zeta_i}}{\sqrt{d}}\right)e^{-2\sqrt{\zeta_i d}t}}. \tag{A21}$$

Note that, for all $0 \leq t \leq D$, the right-hand side of (A21) is always positive and finite. Moreover, from the dynamics of the leading vehicle, we also have that $v_1(t) \geq v_{1_0} + D\min\left\{0, \bar{\delta}_1\left(T_{1_0}, u_{1_0}\right)\right\} > 0$ (by assumption), $t \in [0, D]$. Using (1) we obtain for $0 \leq t \leq D$

$$s_i(t) = s_i(0) + \int_0^t (v_{i-1}(s) - v_i(s))ds. \tag{A22}$$

Thus, using (A20), (A21) we get that for all $0 \leq t \leq D$ the following hold for $i = 1, \ldots, N$

$$\begin{aligned} s_i(t) \geq s_{i_0} + t\big(&m_{i-1}\left(v_{i-1_0}, T_{i-1_0}, u_{i-1_0}\right) \\ &-M_i\left(v_{i_0}, T_{i_0}, u_{i_0}\right)\big), \end{aligned} \tag{A23}$$

where m_{i-1} and M_i are defined in (28) and (30), respectively. Thus, since $s_{i_0} + t(m_{i-1} - M_i) \geq s_{i_0} + D\min\{0, m_{i-1} - M_i\}$, for all $t \in [0, D]$, we have, under condition (33), both that $s_i(t) > 0$ for all $t \in [0, D]$ (and hence, relation (A13) is established), as well as that condition (A15) holds (because from (A21), condition (33) also implies that $c_1 s_i(D) > M_i \geq v_i(D)$). Using (A20), (A21), and the fact that $\bar{\delta}_i \leq T_i(D) \leq \bar{\zeta}_i$, relations (A16) and (A17) are established under assumptions (34) and (35), respectively.

A.2 Proof of stability and string stability

To prove $\mathcal{L}_\infty$ string stability and asymptotic stability, we capitalize on the specific form of the transformed, closed-loop systems given in (A5)–(A7). This is a result of employment of predictor-feedback in combination with the control design procedure developed by Krstic and Bement.[24] We recall the differences of the speed, spacing, and torque states from their respective equilibrium values, which are obtained for a constant equilibrium for speed states (dictated by a constant leader's speed), say v^*, as $\tilde{s}_i = s_i - s^*$, $\tilde{v}_i = v_i - v^*$, and $\tilde{T}_i = T_i - T^*$, with $s^* = hv^*$, $T^* = \frac{dv^{*2}+g}{c}$, corresponding to an equilibrium control input $u_i^* = T^*$, for vehicles $i = 1, \ldots, N$. It follows from (A5) to (A7) that for $t \geq D$ it holds that[‖]

$$\tilde{z}_{i,1}(t) = e^{-c_1(t-D)}\tilde{z}_{i,1}(D) + \int_D^t e^{-c_1(t-s)}\tilde{z}_{i,2}(s)ds - \int_D^t e^{-c_1(t-s)}\tilde{v}_{i-1}(s)ds \tag{A24}$$

$$\tilde{z}_{i,2}(t) = e^{-c_2(t-D)}\tilde{z}_{i,2}(D) + \frac{1}{c_2 - c_3}\left(e^{-c_3(t-D)} - e^{-c_2(t-D)}\right)z_{i,3}(D) - c_1\int_D^t e^{-c_2(t-s)}\tilde{v}_{i-1}(s)ds \tag{A25}$$

$$z_{i,3}(t) = e^{-c_3(t-D)}z_{i,3}(D), \tag{A26}$$

where we used definitions

$$\tilde{z}_{i,1} = z_{i,1} + hv^* \tag{A27}$$
$$\tilde{z}_{i,2} = z_{i,2} - v^* + c_1 hv^*, \tag{A28}$$

and the fact that the $\tilde{z}_{i,1}$, $\tilde{z}_{i,2}$ states also satisfy (A5)–(A7). From (A24)–(A26) it follows by induction (starting from $i = 1$ with $\tilde{v}_0 \equiv 0$, for a constant leading vehicle's speed) that the closed-loop system is asymptotically stable, provided that $\tilde{z}_{i,1}(D)$, $\tilde{z}_{i,2}(D)$, and $\tilde{z}_{i,3}(D)$ are bounded. While for a constant, leading vehicle's speed, that is, $\tilde{v}_0 \equiv 0$, we conclude combining (A2), (A3) with (A27), (A28) that tracking of the desired speed and spacing is achieved. We derive next estimates on solutions for the system in $z_{i,j}$ variables during open-loop operation. Towards this end, it turns out that it is more straightforward to utilize system (1)–(3) in the original variables. The reason is that, using (A2)–(A4), it can then be shown that $\tilde{z}_{i,1}$, $\tilde{z}_{i,2}$, and $z_{i,3}$ remain bounded during open-loop operation, provided that $\tilde{s}_i$, $\tilde{v}_i$, and $\tilde{T}_i$ remain bounded. We start estimating $\tilde{v}_i(t)$ for $t \in [0, D]$, re-writing (2), (3) as

$$\dot{\tilde{v}}_i(t) = -d\tilde{v}_i(t)^2 - 2d\tilde{v}_i(t)v^* + c\tilde{T}_i(t) \tag{A29}$$
$$\dot{\tilde{T}}_i(t) = -\frac{1}{\tau}\tilde{T}_i(t) + \frac{1}{\tau}\tilde{u}_{i_0}(t - D), \tag{A30}$$

where $\tilde{u}_{i_0}(t-D) = u_{i_0}(t-D) - T^*$. Defining $r_i(t) = |\tilde{v}_i(t)| + \sqrt{\frac{c}{d}\sup_{\theta\in[0,D]}\left|\tilde{T}_i(\theta)\right|}$, we obtain from (A29) that

$$\frac{1}{d}\dot{r}_i(t) \leq \tilde{v}_i(t)^2 - 2|\tilde{v}_i(t)|v^* + \frac{c}{d}\sup_{\theta\in[0,D]}\left|\tilde{T}_i(\theta)\right|. \tag{A31}$$

Hence, with $r_i(t) = |\tilde{v}_i(t)| + \sqrt{\frac{c}{d}\sup_{\theta\in[0,D]}\left|\tilde{T}_i(\theta)\right|}$ we get

$$\dot{r}_i(t) \leq dr_i(t)^2. \tag{A32}$$

Thus, under (37), from the comparison principle we get

$$|\tilde{v}_i(t)| \leq \frac{r_{i_0}}{1 - dr_{i_0}t}, \tag{A33}$$

where

$$r_{i_0} = \left|\tilde{v}_{i_0}\right| + \sqrt{\frac{c}{d}\sup_{\theta\in[0,D]}\left|\tilde{T}_i(\theta)\right|}. \tag{A34}$$

Using (A30) we obtain that $\tilde{T}_i(t) = \mathrm{e}^{-\frac{t}{\tau}}\tilde{T}_{i_0} + \frac{1}{\tau}\int_0^t \mathrm{e}^{-\frac{t-s}{\tau}}\tilde{u}_{i_0}(s-D)ds$, and hence,

$$\sup_{t\in[0,D]}\left|\tilde{T}_i(t)\right| \leq \left|\tilde{T}_{i_0}\right| + \sup_{\theta\in[-D,0]}\left|\tilde{u}_{i_0}(\theta)\right|. \tag{A35}$$

Thus, combining (A33) with (A35) we arrive at

$$\sup_{t\in[0,D]}|\tilde{v}_i(t)| \leq \frac{\tilde{r}_{i_0}}{1 - d\tilde{r}_{i_0}D}, \tag{A36}$$

where

$$\tilde{r}_{i_0} = \left|\tilde{v}_{i_0}\right| + \sqrt{\frac{c}{d}}\sqrt{\left|\tilde{T}_{i_0}\right| + \sup_{\theta\in[-D,0]}\left|\tilde{u}_{i_0}(\theta)\right|}. \tag{A37}$$

We next estimate $\tilde{s}_i(D)$. Using the fact that $\dot{\tilde{s}}_i(t) = \tilde{v}_{i-1}(t) - \tilde{v}_i(t)$ we get from (A36) that for $t \in [0, D]$ and $i = 2, \dots, N$ it holds that

$$\sup_{t\in[0,D]} |\tilde{s}_i(t)| \leq \left|\tilde{s}_{i_0}\right| + \frac{D\tilde{r}_{i_0}}{1 - d\tilde{r}_{i_0}D} + \frac{D\tilde{r}_{i-1_0}}{1 - d\tilde{r}_{i-1_0}D}, \tag{A38}$$

whereas for $i = 1$ it holds that

$$\sup_{t\in[0,D]} |\tilde{s}_i(t)| \leq \left|\tilde{s}_{i_0}\right| + \frac{D\tilde{r}_{i_0}}{1 - d\tilde{r}_{i_0}D} + D\left|\tilde{v}_{1_0}\right| + D^2\left|\tilde{T}_{1_0}\right| + D^2 \sup_{\theta\in[-D,0]} \left|\tilde{u}_{1_0}(\theta)\right|. \tag{A39}$$

From definitions (A2)–(A4) it follows that

$$\left|\tilde{z}_{i,1}(D)\right| \leq |\tilde{s}_i(D)| \tag{A40}$$

$$\left|\tilde{z}_{i,2}(D)\right| \leq |\tilde{v}_i(D)| + c_1|\tilde{s}_i(D)| \tag{A41}$$

$$\left|\tilde{z}_{i,3}(D)\right| \leq c\left|\tilde{T}_i(D)\right| + (c_1 + c_2 + 2dv^*)|\tilde{v}_i(D)| + d|\tilde{v}_i(D)|^2 + c_1c_2|\tilde{s}_i(D)|, \tag{A42}$$

for $i = 1, \dots, N$. Thus, asymptotic stability follows from (A35) to (A42) combining (A24)–(A26) with the inverse transformation to (A2)–(A4).

We next establish $\mathcal{L}_\infty$ string stability, estimating $\|\tilde{v}_i\|_\infty$. Using the fact that $\sup_{t\in[0,+\infty)} |\tilde{v}_i(t)| \leq \sup_{t\in[0,D]} |\tilde{v}_i(t)| + \sup_{t\in[D,+\infty)} |\tilde{v}_i(t)|$ and (A36), (A37) it remains to estimate $\sup_{t\in[D,+\infty)} |\tilde{v}_i(t)|$. Since $\tilde{v}_i = \tilde{z}_{i,2} - c_1\tilde{z}_{i,1}$, we obtain from (A24) to (A26) and using triangular inequality that for $t \geq D$ and $i = 1, \dots, N$ the following hold

$$|\tilde{v}_i(t)| \leq k\left(\left|\tilde{z}_{i,1}(D)\right| + \left|\tilde{z}_{i,2}(D)\right| + \left|z_{i,3}(D)\right|\right)e^{-\lambda(t-D)} + c_1c_2\int_D^t \left|\frac{e^{-c_2(t-s)} - e^{-c_1(t-s)}}{c_1 - c_2}\right| ds \sup_{t\geq D}|\tilde{v}_{i-1}(t)|, \tag{A43}$$

for some $k, \lambda > 0$ (dependent on c_1, c_2, c_3), where we used the fact that $c_1^2\int_D^t e^{-c_1(t-s)}\int_D^s e^{-c_2(s-r)}\tilde{v}_{i-1}(r)drds = \frac{c_1^2}{c_1-c_2}\int_D^t e^{-c_2(t-s)}\tilde{v}_{i-1}(s)ds - \frac{c_1^2}{c_1-c_2}\int_D^t e^{-c_1(t-s)}\tilde{v}_{i-1}(s)ds$. Thus, since for all $t \geq s \geq D$ it holds that $\frac{e^{-c_2(t-s)}-e^{-c_1(t-s)}}{c_1-c_2} \geq 0$, we obtain

$$|\tilde{v}_i(t)| \leq k\left(\left|\tilde{z}_{i,1}(D)\right| + \left|\tilde{z}_{i,2}(D)\right| + \left|z_{i,3}(D)\right|\right) + \|\tilde{v}_{i-1}\|_\infty \frac{c_1 - c_2 - c_1e^{-c_2(t-D)} + c_2e^{-c_1(t-D)}}{c_1 - c_2}. \tag{A44}$$

The function $1 - \frac{c_1e^{-c_2(t-D)}-c_2e^{-c_1(t-D)}}{c_1-c_2}$ is increasing for all $t \geq D$, and hence, $1 - \frac{c_1e^{-c_2(t-D)}-c_2e^{-c_1(t-D)}}{c_1-c_2} \leq 1$ for all $t \geq D$. Therefore, it follows from (A44) that for $i = 1, \dots, N$

$$\sup_{t\geq D}|\tilde{v}_i(t)| \leq k\left(\left|\tilde{z}_{i,1}(D)\right| + \left|\tilde{z}_{i,2}(D)\right| + \left|z_{i,3}(D)\right|\right) + \|\tilde{v}_{i-1}\|_\infty, \tag{A45}$$

which, in combination with (A35)–(A42), establishes string stability in $\mathcal{L}_\infty$. In particular, using (A36) we arrive at

$$\sup_{t\in[0,+\infty)} |\tilde{v}_i(t)| \leq \frac{\tilde{r}_{i_0}}{1 - d\tilde{r}_{i_0}D} + \sup_{t\in[D,+\infty)} |\tilde{v}_i(t)|, \tag{A46}$$

and hence, combining (A46) with (A45) and (A40)–(A42), we obtain (22) with

$$\gamma_0(y) = y \tag{A47}$$

$$\gamma_1(y) = k(1 + c_1 + c_1c_2)y \tag{A48}$$

$$\gamma_2(y) = (1 + kD + kc_1 + kc_1D + kc_1c_2D + kc_2 + 2kdv^* + k)\frac{y}{1 - dyD} + kd\left(\frac{y}{1 - dyD}\right)^2 + dky^2 \tag{A49}$$

$$\gamma_3(y) = k(1 + c_1 + c_1c_2)D\left(\frac{y}{1 - dyD} + y + \frac{Dd}{c}y^2\right), \tag{A50}$$

using (A35)–(A39).