Final exam: SYS401

Instructor: Nikos Bekiaris-Liberis

Date and place: January 15 2016 in class

Duration: 2 hours and 15 minutes

Open notes

Not clearly presented answers will not be graded

Collaboration is not allowed

Problem 1 (Bode diagram): Figure 1 (blue line) shows the asymptotic Bode diagram of the magnitude of transfer function

$$G(s) = K\frac{(\tau_1 s+1)(\tau_2 s+1)}{s^{p_1}(\tau_3 s+1)^{p_2}}. \tag{1}$$

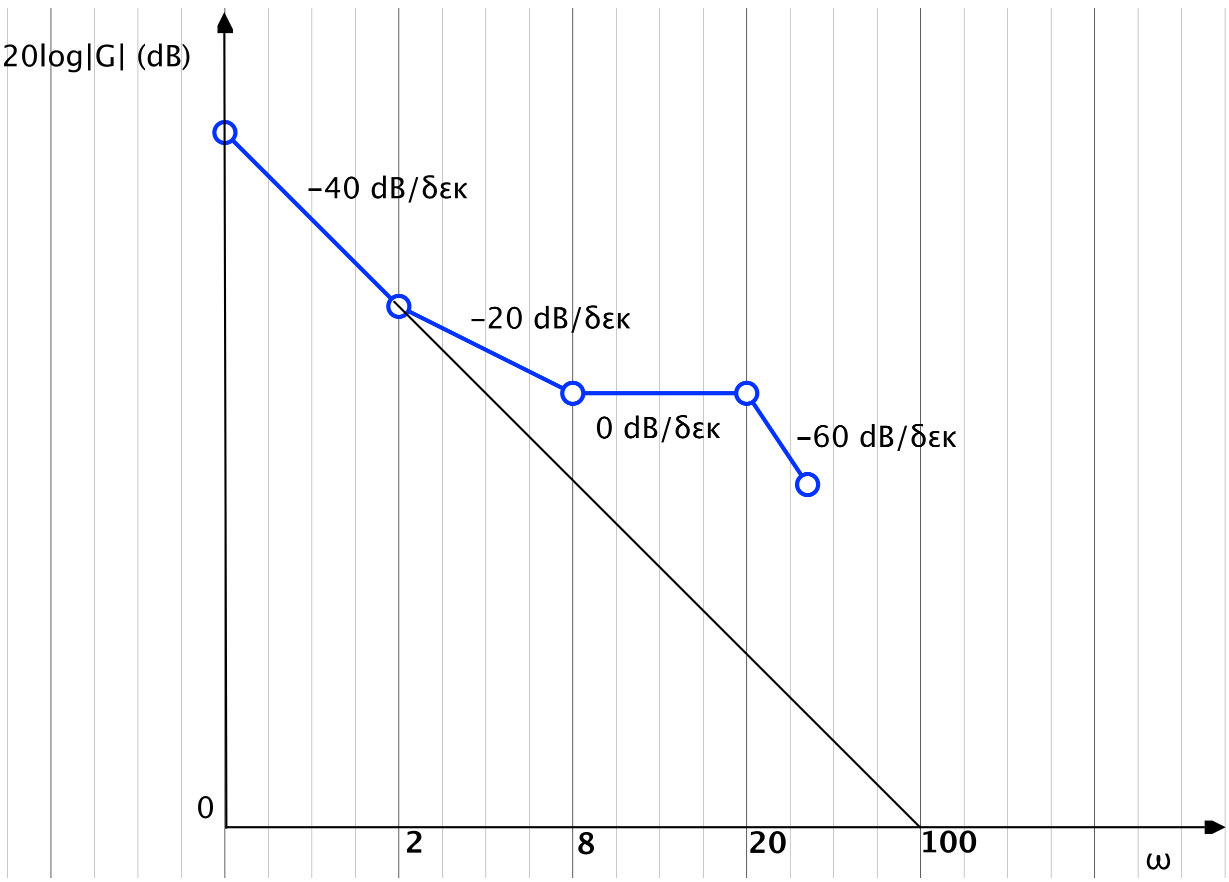

Figure 1: Asymptotic Bode diagram (blue) of the magnitude of (1).

a) Find K, τ_1, τ_2, τ_3, p_1, and p_2. (Units 3.5)

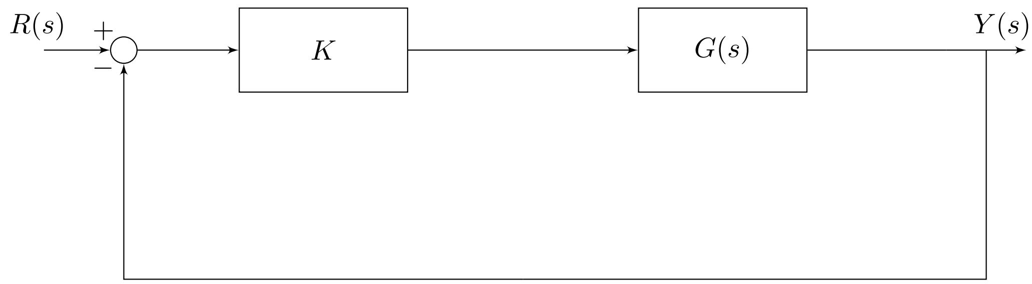

Figure 2: Block diagram of system in Problem 2.

Problem 2 (Nyquist criterion): We consider a system with a block diagram given in Figure 2, where $K > 0$. The Nyquist diagram of G, which has no pole on the right-half complex plane, is shown in Figure 3.

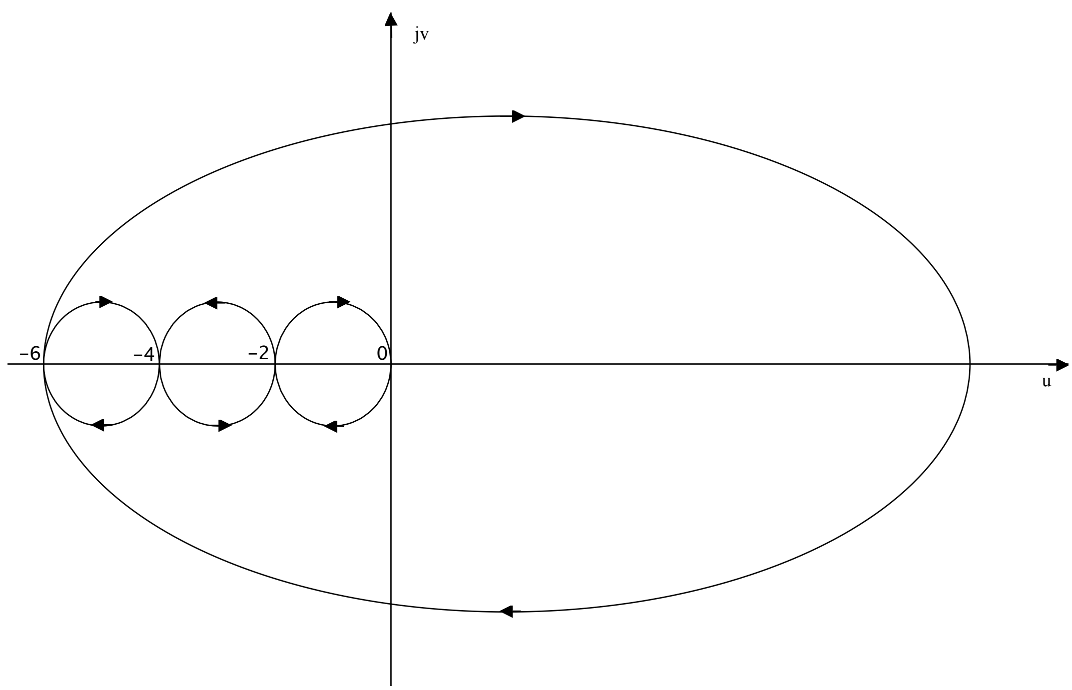

Figure 3: Nyquist diagram of G in Problem 2.

a) Determine the values of $K > 0$ such that the closed-loop system is stable. (Units 2.5)

b) For the values of $K > 0$ for which the closed-loop system is unstable, find the number of the unstable roots of the characteristic equation of the closed-loop system. (Unit 1)

Problem 3 (Root locus): For the block diagram in Figure 2 assume that

$$G(s) = \frac{1}{(s+1)^2}, \tag{2}$$

where $K \leq 0$.

a) Given that for $K \leq 0$ the parts of the root locus on the real axis are located on the left of an even number of poles and zeros, sketch the root locus of the closed-loop system for $K \leq 0$. (Unit 1)

b) Find the points at which the locus crosses the imaginary axis and the values of $K \leq 0$ when this happens. (Unit 0.5)

c) Consider a system with the block diagram of Figure 2, where $K \leq 0$ and G given by

$$G(s) = \frac{\prod_{i=1}^{m}(s^2 + 2\zeta_i {\omega_n}_i s + {\omega_n}_i^2)}{s^N \prod_{i=m+1}^{m+M}(s^2 + 2\zeta_i {\omega_n}_i s + {\omega_n}_i^2)}, \tag{3}$$

with $0 < \zeta_i < 1$, $i = 1, 2, \ldots, m+M$. Show that for any N, all points on the positive real axis belong to the root locus of the closed-loop system. If in addition N is even, show that any point on the negative real axis belongs to the root locus of the closed-loop system as well. (Units 1.5)