Problem Set 1: SYS401

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Due: Oct 23, 2015 **in class**

Problem 1 (State-space representation, time response): Consider the following system

$$\dot{x}_1(t) = -x_1(t) + r(t) \tag{1}$$
$$\dot{x}_2(t) = 2x_1(t) - 3x_2(t) + r(t) + d(t), \tag{2}$$

where r is the control input and d is a disturbance input. The output of the system is $y = x_1$.

a) Write system (1), (2) in state-space form, i.e., determine A, B_r, B_d, and C such that

$$\dot{x}(t) = Ax(t) + B_r r(t) + B_d d(t) \tag{3}$$
$$y(t) = Cx(t), \tag{4}$$

where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

b) Determine the state-transition matrix e^{At}.

c) Find the response of the system $x(t)$ for an initial condition $x(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and zero inputs, i.e., $r(t) = d(t) = 0$, $\forall t \geq 0$.

d) Find the transfer function $G_r(s) = \frac{Y(s)}{R(s)}$. (Assume zero initial conditions and zero disturbance input.)

e) Find the transfer function $G_d(s) = \frac{Y(s)}{D(s)}$. (Assume zero initial conditions and zero control input.)

f) Using d) find the response of the system to a unit step control input, i.e., $u(t) = 1$, $\forall t \geq 0$.

g) Using e) find the response of the system to a unit step disturbance input, i.e., $d(t) = 1$, $\forall t \geq 0$.

Problem 2 (Transfer function, feedback): Consider a system represented by the block diagram shown in Figure 1, where $a, K > 0$.

a) Determine the transfer function $G_R = \frac{Y(s)}{R(s)}$. (Assume $D(s) = 0$.)

b) Determine the transfer function $G_D = \frac{Y(s)}{D(s)}$. (Assume $R(s) = 0$.)

c) Find the sensitivity $S_a^{G_R}$.

d) Find the steady-state error $e_r = \lim_{t\to\infty} (y(t) - r(t))$ when $R(s) = \frac{1}{s}$ and $D(s) = 0$.

e) Find the steady-state error $e_d = \lim_{t\to\infty} (y(t) - r(t))$ when $R(s) = 0$ and $D(s) = \frac{1}{s}$.

f) Find the values of K such that $|e_d| \leq 0.01$.

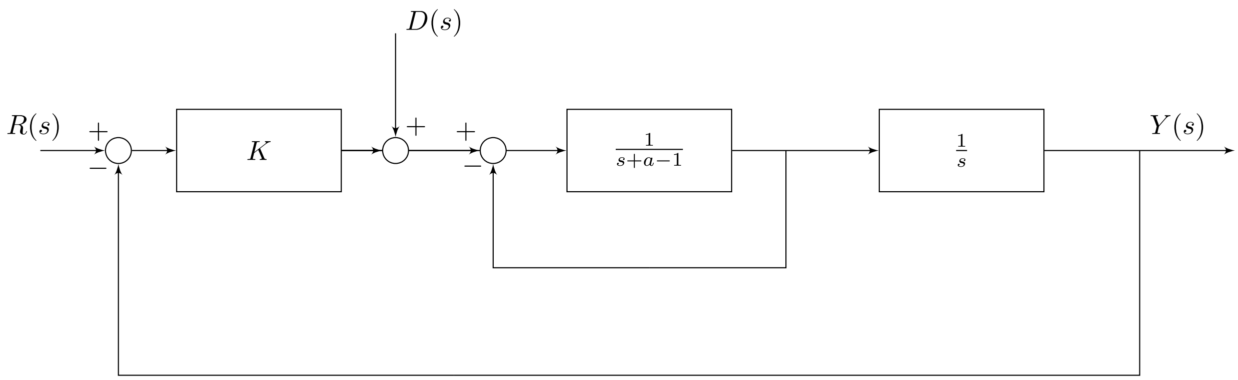

Figure 1: Block diagram of the system in Problem 2.

g) For $a = 101$ and $K = 100$ find $y(t)$ when $r(t) = 1$ and $d(t) = 0.1$, $\forall t \geq 0$.

BONUS:

Problem 3 (Equilibria, linearization): Consider the following system

$$\dot{x}_1(t) = x_1(t)\,(1 - x_1(t) - a x_2(t)) \tag{5}$$

$$\dot{x}_2(t) = b x_2(t)\,(x_1(t) - x_2(t))\,, \tag{6}$$

where x_1 and x_2 represent the pray and predator populations, respectively, and $a, b > 0$. This system has three different equilibria (i.e., constant solutions), one corresponding to the case where neither the prays nor the predators survive, one where only prays survive, and one where both populations survive.

a) Determine the equilibria of the system.

b) Linearize the system around all three equilibria.