SINGLE-CARRIER SYSTEMS WITH MMSE LINEAR EQUALIZERS: PERFORMANCE DEGRADATION DUE TO CHANNEL AND CFO ESTIMATION ERRORS

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ABSTRACT

We assess the impact of the channel and carrier frequency offset (CFO) estimation errors on the performance of single-carrier systems with MMSE linear equalizers. Performance degradation is caused by the fact that a *mismatched* MMSE linear equalizer is applied to channel output samples with *imperfectly canceled* CFO. We develop asymptotic expressions for the excess mean square error (EMSE) induced by the channel and CFO estimation errors. Under some realistic assumptions, we derive a simple EMSE approximation which reveals that performance degradation is mainly caused by the imperfectly canceled CFO. Furthermore, the EMSE is approximately proportional to the CFO estimation error variance, with the proportionality factor being independent of the training sequence. Thus, optimal training sequence (TS) design for CFO estimation is also highly relevant for *joint* channel and CFO estimation.

Index Terms— joint channel and CFO estimation, linear equalization.

1. INTRODUCTION

A problem that frequently arises in packet-based wireless systems is the *joint* estimation of the frequency selective channel and the CFO [1]. Optimal TS design for this problem has been considered in [2]. The optimized cost function in [2] was the *asymptotic* Cramér-Rao bound (CRB). However, the channel and the CFO estimation errors were assigned equal weight. This might be *suboptimal* since "*... presumably channel estimation errors will have a different impact, e.g., on bit-error rate, than frequency estimation errors*" [2]. It seems that the *unequal weighting* problem cannot be resolved unless one considers specific receiver structures [3].

In this work, we consider a receiver with an MMSE linear equalizer. Performance degradation is caused by the fact that a *mismatched* MMSE linear equalizer is applied to channel output samples with *imperfectly canceled* CFO. In order to uncover the relative importance of these error sources, we develop an asymptotic expression for the induced EMSE which, however, is very complicated. Under a *small ideal MMSE* assumption, we derive a simple and informative EMSE approximation which reveals that the most important error source is the imperfectly canceled CFO. Furthermore, the EMSE is approximately proportional to the CFO estimation error variance, with the proportionality factor being *independent* of the TS. Thus, optimal TS design for CFO estimation is also highly relevant for *joint* channel and CFO estimation. We also highlight the fact that the placement of the TS at the middle of the transmitted packet leads to smaller EMSE.

Notation: Superscripts ^{T}, ^{H} and * denote transpose, conjugate transpose and elementwise conjugation, respectively. $\mathrm{Re}\{\cdot\}$ denotes the real part of a complex number. $\mathbf{P}_{\mathcal{R}(\mathbf{A})}$ and $\mathbf{P}^{\perp}_{\mathcal{R}(\mathbf{A})}$ denote the orthogonal projectors onto the column space of matrix $\mathbf{A}$ and onto its orthogonal complement.

2. CHANNEL AND CFO ESTIMATION

2.1. The channel model

We consider the linear baseband-equivalent discrete-time frequency-selective channel with output

$$z_n = \sum_{l=0}^{L} h_l a_{n-l} + w_n \tag{1}$$

where a_n and w_n denote the channel input and additive channel noise, respectively. The input packet has length N. The input symbols are i.i.d. unit variance circular. The noise samples are i.i.d. circular Gaussian, with variance σ_w^2. The channel impulse response is $\mathbf{h} \triangleq [h_0 \cdots h_L]^T$. The channel output vector $\mathbf{z}_{n:n-M} \triangleq [z_n \ \cdots \ z_{n-M}]^T$ can be expressed as

$$\mathbf{z}_{n:n-M} = \mathbf{H}\mathbf{a}_{n:n-M-L} + \mathbf{w}_{n:n-M} \tag{2}$$

where $\mathbf{H}$ is the $(M+1) \times (M+L+1)$ Toeplitz filtering matrix constructed by $\mathbf{h}$.

If angular CFO ω is present, then the channel output is

$$r_n = \mathrm{e}^{j\omega n} \sum_{l=0}^{L} h_l a_{n-l} + w_n \tag{3}$$

and, similarly to (2), we can write

$$\mathbf{r}_{n:n-M} = \mathbf{\Gamma}_{n:n-M}(\omega)\mathbf{H}\mathbf{a}_{n:n-L-M} + \mathbf{w}_{n:n-M} \qquad (4)$$

where $\mathbf{\Gamma}_{n:n-M}(\omega) \triangleq \mathrm{diag}(\mathrm{e}^{j\omega n}, \ldots, \mathrm{e}^{j\omega(n-M)})$.

2.2. Channel and CFO estimation

We assume that the N_{tr} symbols $\mathbf{a}_{\mathrm{tr}} \triangleq [a_{n_1} \ \cdots \ a_{n_2}]^T$, with $N_{\mathrm{tr}} \triangleq n_2 - n_1 + 1$, are used for training. Collecting the output samples that depend *only* on the training, we obtain

$$\mathbf{y} \triangleq \mathbf{r}_{n_2:n_1+L} = \mathbf{\Gamma}_{n_2:n_1+L}(\omega)\mathbf{A}\mathbf{h} + \mathbf{w}_{n_2:n_1+L} \qquad (5)$$

where $\mathbf{A}$ is the $(N_{\mathrm{tr}} - L) \times (L+1)$ Hankel matrix

$$\mathbf{A} \triangleq \begin{bmatrix} a_{n_2} & \cdots & a_{n_2-L} \\ \vdots & \ddots & \vdots \\ a_{n_1+L} & \cdots & a_{n_1} \end{bmatrix}. \qquad (6)$$

It turns out [4] that the estimate of $\mathbf{h}$ and, thus, its accuracy, depends on n_1, n_2 and $\hat{\omega}$ through $\mathbf{\Gamma}_{n_2:n_1+L}(\hat{\omega})$. An accurate channel estimate[1] is obtained if we rewrite (5) as

$$\mathbf{y} = \mathbf{\Gamma}_{\frac{N_{\mathrm{tr}}-L}{2}-1:-\frac{N_{\mathrm{tr}}-L}{2}}(\omega)\mathbf{A}\mathbf{h}' + \mathbf{w}_{n_2:n_1+L} \qquad (7)$$

where $\mathbf{h}' \triangleq \mathrm{e}^{j\omega\xi}\mathbf{h}$ and $\xi \triangleq n_1 + \frac{N_{\mathrm{tr}}+L}{2}$, i.e., ξ is the middle position of $\mathbf{y}$, and instead of $\mathbf{h}$ we estimate $\mathbf{h}'$ (for details see [4]). The joint ML estimates of ω and $\mathbf{h}'$ are [1]

$$\hat{\omega} = \underset{\tilde{\omega}}{\mathrm{argmax}}\{\mathbf{y}^H\mathbf{\Gamma}_{n_2:n_1+L}(\tilde{\omega})\mathbf{P}_{\mathcal{R}(\mathbf{A})}\mathbf{\Gamma}^H_{n_2:n_1+L}(\tilde{\omega})\mathbf{y}\} \qquad (8)$$

$$\hat{\mathbf{h}}' = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{\Gamma}^H_{\frac{N_{\mathrm{tr}}-L}{2}-1:-\frac{N_{\mathrm{tr}}-L}{2}}(\hat{\omega})\,\mathbf{y}. \qquad (9)$$

If we define $\Delta\omega \triangleq \hat{\omega} - \omega$, $\Delta\mathbf{h}' \triangleq \hat{\mathbf{h}}' - \mathbf{h}'$, and

$$\mathbf{K}' \triangleq \mathrm{diag}\Big(\frac{N_{\mathrm{tr}}-L}{2} - 1, \ldots, -\frac{N_{\mathrm{tr}}-L}{2}\Big),$$

then the *finite sample* CRBs [2] imply that

$$\sigma^2_{\Delta\omega} \triangleq \mathcal{E}\left((\Delta\omega)^2\right) = \frac{1}{2}\,\sigma^2_w\left[\mathrm{tr}\left(\mathbf{h}^H\mathbf{A}^H\mathbf{K}'\mathbf{P}^{\perp}_{\mathcal{R}(\mathbf{A})}\mathbf{K}'\mathbf{A}\mathbf{h}\right)\right]^{-1} \qquad (10)$$

$$\mathbf{C}' \triangleq \mathcal{E}\left(\Delta\mathbf{h}'\Delta\mathbf{h}'^H\right) = \sigma^2_w(\mathbf{A}^H\mathbf{A})^{-1} + \sigma^2_{\Delta\omega}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{K}'\mathbf{A}\mathbf{h}'\mathbf{h}'^H\mathbf{A}^H\mathbf{K}'\mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1} \qquad (11)$$

$$\mathbf{C}'_t \triangleq \mathcal{E}\left(\Delta\mathbf{h}'\Delta\mathbf{h}'^T\right) = -\sigma^2_{\Delta\omega}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{K}'\mathbf{A}\mathbf{h}'\mathbf{h}'^T \times \mathbf{A}^T\mathbf{K}'\mathbf{A}^*(\mathbf{A}^H\mathbf{A})^{-T} \qquad (12)$$

and

$$\mathcal{E}\left(\Delta\omega\Delta\mathbf{h}'\right) = j\sigma^2_{\Delta\omega}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{K}'\mathbf{A}\mathbf{h}'. \qquad (13)$$

We assume that the noise variance σ^2_w is known at the receiver.

[1] However, we do not claim optimality, in general.

3. THE MISMATCHED MMSE LINEAR EQUALIZER

3.1. The ideal case

The order-M delay-d MMSE linear equalizer is [5]

$$\mathbf{f} = \left(\mathbf{H}'\mathbf{H}'^H + \sigma^2_w\mathbf{I}_{M+1}\right)^{-1}\mathbf{H}'\mathbf{e}_d = \mathbf{R}_{\mathbf{z}}^{-1}\mathbf{H}'\mathbf{e}_d \qquad (14)$$

where $\mathbf{H}'$ is the filtering matrix constructed by $\mathbf{h}'$, $\mathbf{R}_{\mathbf{z}} \triangleq \mathcal{E}_{a,w}\left[\mathbf{z}_{n:n-M}\mathbf{z}^H_{n:n-M}\right] = \mathbf{H}'\mathbf{H}'^H + \sigma^2_w\mathbf{I}_{M+1}$ and $\mathbf{e}_d$ is the $(M+L+1)\times 1$ vector with 1 at the $(d+1)$-st position and zeros elsewhere. The corresponding MMSE is

$$\mathrm{MMSE} = 1 - \mathbf{f}^H\mathbf{R}_{\mathbf{z}}\mathbf{f}. \qquad (15)$$

For later use, we define $\mathbf{R}$ as the $(M+1)\times(L+1)$ Hankel matrix constructed by $\mathbf{r} \triangleq \mathbf{c} - \mathbf{e}_d$, with $\mathbf{c}$ being the combined (channel-equalizer) impulse response, that is, $\mathbf{c} \triangleq \mathbf{H}'^T\mathbf{f}^*$, and $\mathbf{G} \triangleq \mathbf{H}'\mathbf{F}^T$, where $\mathbf{F}$ is the $(L+1)\times(L+M+1)$ Toeplitz filtering matrix constructed by $\mathbf{f}$.

3.2. CFO correction and mismatched MMSE equalizer

Adopting the channel model presented in (7), the channel output is expressed as

$$r'_n = \mathrm{e}^{j\omega(n-\xi)}\sum_{l=0}^{L} h'_l a_{n-l} + w_n. \qquad (16)$$

After the computation of $\hat{\omega}$, we proceed to CFO correction

$$s'_n = \mathrm{e}^{-j\hat{\omega}(n-\xi)}r'_n. \qquad (17)$$

Then, it can be shown that $\mathbf{s}'_{n:n-M}$ can be expressed as

$$\mathbf{s}'_{n:n-M} = \mathrm{e}^{j\Delta\omega\xi}\mathbf{\Gamma}_{n:n-M}(-\Delta\omega)\mathbf{H}'\mathbf{a}_{n:n-L-M} + \mathrm{e}^{j\hat{\omega}\xi}\mathbf{\Gamma}_{n:n-M}(-\hat{\omega})\mathbf{w}_{n:n-M}. \qquad (18)$$

If we use in (14) the channel estimate $\hat{\mathbf{h}}'$ as if it were the true channel $\mathbf{h}'$, we compute the mismatched MMSE equalizer

$$\hat{\mathbf{f}} = \left(\hat{\mathbf{H}}'\hat{\mathbf{H}}'^H + \sigma^2_w\mathbf{I}_{M+1}\right)^{-1}\hat{\mathbf{H}}'\mathbf{e}_d. \qquad (19)$$

The equalizer mismatch is defined as $\Delta\mathbf{f} \triangleq \hat{\mathbf{f}} - \mathbf{f}$. The input symbol estimation error at the time instant n is

$$\hat{e}_n = \hat{\mathbf{f}}^H\mathbf{s}'_{n:n-M} - \mathbf{e}^H_d\mathbf{a}_{n:n-L-M} \qquad (20)$$

and the mean square estimation error is

$$\begin{aligned}\mathrm{MSE}_n(\hat{\mathbf{f}}, \hat{\omega}) &\triangleq \mathcal{E}_{a,w}\left[|\,\hat{e}_n|^2\right] \\ &= \hat{\mathbf{f}}^H\Big(\mathbf{\Gamma}_{n:n-M}(-\Delta\omega)\mathbf{H}'\mathbf{H}'^H\mathbf{\Gamma}^H_{n:n-M}(-\Delta\omega) + \sigma^2_w\mathbf{I}_{M+1}\Big)\hat{\mathbf{f}} \\ &\quad - 2\,\mathrm{Re}\{\mathrm{e}^{j\Delta\omega\xi}\,\hat{\mathbf{f}}^H\mathbf{\Gamma}_{n:n-M}(-\Delta\omega)\mathbf{H}'\mathbf{e}_d\} + 1.\end{aligned}$$

We observe that the mean square estimation error is time-dependent.

4. EMSE ANALYSIS

The EMSE at the time instant n is defined as

$$\mathrm{EMSE}_n(\hat{\mathbf{f}},\hat{\omega}) \triangleq \mathcal{E}_{\Delta\mathbf{h}',\Delta\omega}[\mathrm{MSE}_n(\hat{\mathbf{f}},\hat{\omega})] - \mathrm{MMSE}. \quad (21)$$

Proposition 1. *The EMSE induced by the channel and CFO estimation errors at time instant n, for $n \in \mathcal{D} \triangleq \{d+1,\ldots,n_1+d-1\} \cup \{n_2+d+1,\ldots,N+d\}$,[2] can be approximated as*

$$\mathrm{EMSE}_n(\hat{\mathbf{f}},\hat{\omega}) \approx \mathbf{T}_1 + \mathbf{T}_2(n) + \mathbf{T}_3(n) \quad (22)$$

where

$$\mathbf{T}_1 \triangleq \mathrm{tr}\Big(\mathbf{R}_z^{-1}\big(\mathbf{R}^*\mathbf{C}'\mathbf{R}^T + \mathbf{G}\mathbf{C}'^*\mathbf{G}^H + \mathbf{G}\mathbf{C}_t'^*\mathbf{R}^T + \mathbf{R}^*\mathbf{C}_t'\mathbf{G}^H\big)\Big) \quad (23a)$$

$$\mathbf{T}_2(n) \triangleq \sigma_{\Delta\omega}^2 \mathrm{Re}\{\mathbf{f}^H\mathbf{D}'^2_{n:n-M}\mathbf{H}'\mathbf{e}_d\} \quad (23b)$$

$$\mathbf{T}_3(n) \triangleq 2\sigma_{\Delta\omega}^2 \mathrm{Re}\big\{\mathbf{h}'^H\mathbf{A}^H\mathbf{K}'\mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1} \times \mathbf{R}^T\mathbf{R}_z^{-1}\mathbf{D}'_{n:n-M}\mathbf{H}'\mathbf{e}_d - \mathbf{h}'^T\mathbf{A}^T\mathbf{K}'\mathbf{A}^*(\mathbf{A}^H\mathbf{A})^{-T} \times \mathbf{G}^H\mathbf{R}_z^{-1}\mathbf{D}'_{n:n-M}\mathbf{H}'\mathbf{e}_d\big\} \quad (23c)$$

and $\mathbf{D}'_{n:n-M} \triangleq \mathrm{diag}((n-\xi),\ldots,(n-M-\xi))$.

Proof: The details of the approximation and the proof are provided in [4]. □

Term $\mathbf{T}_1$ involves only the channel estimation error second-order statistics. In fact, it is the EMSE that would result if the mismatched equalizer were applied to perfectly CFO-corrected channel output samples [5, eq. (28)]. Term $\mathbf{T}_2(n)$ involves only the CFO estimation error variance, and is the EMSE that would result if the ideal MMSE equalizer were applied to imperfectly corrected data. Term $\mathbf{T}_3(n)$ involves both the channel and CFO estimation errors.

4.1. "Small ideal MMSE" assumption

In order to be able to derive insightful EMSE approximations we assume that the *ideal* MMSE is sufficiently small, i.e., the equalizer length is sufficiently large, the SNR is sufficiently high and the delay is chosen carefully. This assumption defines a scenario of high practical importance because it refers to the cases where the MMSE linear equalizer seems most suitable. Under this assumption, matrix $\mathbf{R}$ becomes "small" with respect to $\mathbf{G}$ (both matrices are defined after (15)).[3] Consequently, $\mathbf{T}_1$ and $\mathbf{T}_3(n)$ can be approximated as

$$\mathbf{T}_1 \approx \mathrm{tr}\left(\mathbf{R}_z^{-1}\mathbf{G}\mathbf{C}'^*\mathbf{G}^H\right) \quad (24)$$

$$\mathbf{T}_3(n) \approx -2\sigma_{\Delta\omega}^2 \mathrm{Re}\big\{\mathbf{h}'^T\mathbf{A}^T\mathbf{K}'\mathbf{A}^*(\mathbf{A}^H\mathbf{A})^{-T} \times \mathbf{G}^H\mathbf{R}_z^{-1}\mathbf{D}'_{n:n-M}\mathbf{H}'\mathbf{e}_d\big\}. \quad (25)$$

[2] We do not compute the EMSE for the training symbols a_n, $n = n_1,\ldots,n_2$.

[3] see the discussion before eq. (30) of [5].

4.1.1. Time-average EMSE

Significant insight can be gained if we study the EMSE time-average, across the time instances that correspond to the unknown transmitted data, defined as [3]

$$\begin{aligned}\mathrm{EMSE}(\hat{\mathbf{f}},\hat{\omega}) &\triangleq \frac{1}{n_1-1}\sum_{n=d+1}^{n_1+d-1}\mathrm{EMSE}_n(\hat{\mathbf{f}},\hat{\omega}) + \frac{1}{N-n_2}\sum_{n=n_2+d+1}^{N+d}\mathrm{EMSE}_n(\hat{\mathbf{f}},\hat{\omega}) \\ &= \mathbf{T}_1 + \frac{1}{n_1-1}\sum_{n=d+1}^{n_1+d-1}(\mathbf{T}_2(n)+\mathbf{T}_3(n)) + \frac{1}{N-n_2}\sum_{n=n_2+d+1}^{N+d}(\mathbf{T}_2(n)+\mathbf{T}_3(n)).\end{aligned}$$

If we define $\mathcal{C}_1 \triangleq \frac{1}{n_1-1}\sum_{n=d+1}^{n_1+d-1} n^2 + \frac{1}{N-n_2}\sum_{n=n_2+d+1}^{N+d} n^2$ and $\mathcal{C}_2 \triangleq \frac{1}{n_1-1}\sum_{n=d+1}^{n_1+d-1} n + \frac{1}{N-n_2}\sum_{n=n_2+d+1}^{N+d} n$, then it is easy to show that

$$\begin{aligned}\mathbf{T}_2 &\triangleq \frac{1}{n_1-1}\sum_{n=d+1}^{n_1+d-1}\mathbf{T}_2(n) + \frac{1}{N-n_2}\sum_{n=n_2+d+1}^{N+d}\mathbf{T}_2(n) \\ &= \sigma_{\Delta\omega}^2\Big[\underbrace{\left(\mathcal{C}_1 - 2\,\mathcal{C}_2\xi + 2\xi^2\right)\mathrm{Re}\{\mathbf{f}^H\mathbf{H}'\mathbf{e}_d\}}_{\mathbf{T}_{21}} \\ &\quad \underbrace{-2(\mathcal{C}_2-2\xi)\mathrm{Re}\{\mathbf{f}^H\mathbf{D}_M\mathbf{H}'\mathbf{e}_d\}}_{\mathbf{T}_{22}}\underbrace{+2\mathrm{Re}\{\mathbf{f}^H\mathbf{D}_M^2\mathbf{H}'\mathbf{e}_d\}}_{\mathbf{T}_{23}}\Big]\end{aligned}$$

and

$$\begin{aligned}\mathbf{T}_3 &\triangleq \frac{1}{n_1-1}\sum_{n=d+1}^{n_1+d-1}\mathbf{T}_3(n) + \frac{1}{N-n_2}\sum_{n=n_2+d+1}^{N+d}\mathbf{T}_3(n) \\ &\approx -2\sigma_{\Delta\omega}^2\,\mathrm{Re}\Big\{\mathbf{h}'^T\mathbf{A}^T\mathbf{K}'\mathbf{A}^*(\mathbf{A}^H\mathbf{A})^{-T}\mathbf{G}^H\mathbf{R}_z^{-1} \\ &\quad \times \Big(\underbrace{(\mathcal{C}_2 - 2\,\xi)}_{t_{31}}\,\mathbf{I}_{M+1} - 2\,\mathbf{D}_M\Big)\mathbf{H}'\mathbf{e}_d\Big\}.\end{aligned}$$

Both $\mathbf{T}_2$ and $\mathbf{T}_3$ depend on ξ. It turns out that there does *not* exist a *unique channel independent* ξ that is optimal, i.e., always attains minimum EMSE. If we put $\xi = \frac{\mathcal{C}_2}{2}$,[4] then term $\mathbf{T}_{21}$ is minimized[5] and terms $\mathbf{T}_{22}$ and t_{31} vanish. In the sequel, we adopt this simple choice (however, we do not claim optimality, in general). Then, if we define

$$\mathcal{C} \triangleq \left(\mathcal{C}_1 - \frac{\mathcal{C}_2^2}{2}\right) \quad (26)$$

[4] This implies that the training block is placed close to the middle of the packet (see the definition of ξ after (7)).

[5] We shall see that $\mathbf{T}_{21}$ is the most significant EMSE term.

we obtain

$$\mathbf{T}_2 = \sigma^2_{\Delta\omega} \left[\mathcal{C}\, \mathrm{Re}\{\mathbf{f}^H \mathbf{H}' \mathbf{e}_d\} + 2\, \mathrm{Re}\{\mathbf{f}^H \mathbf{D}^2_M \mathbf{H}' \mathbf{e}_d\} \right] \quad (27)$$

and

$$\mathbf{T}_3 \approx 4\, \sigma^2_{\Delta\omega}\, \mathrm{Re} \left\{ \mathbf{h}'^T \mathbf{A}^T \mathbf{K}' \mathbf{A}^* (\mathbf{A}^H \mathbf{A})^{-T} \mathbf{G}^H \mathbf{R}_z^{-1} \mathbf{D}_M \mathbf{H}' \mathbf{e}_d \right\}. \quad (28)$$

Thus, the EMSE time-average is approximately equal to the sum of the three terms in (24), (27) and (28).

It can be shown [4] that if $\mathbf{H}$ and $\mathbf{A}$ are not very ill-conditioned and $\frac{N_{\mathrm{tr}}}{N}$ is sufficiently small, then $\mathbf{T}_2$ dominates both $\mathbf{T}_1$ and $\mathbf{T}_3$. Thus, the EMSE is approximately equal to $\mathbf{T}_2$ and performance degradation is mainly caused by the imperfectly canceled CFO. Furthermore, a simplified approximate expression for $\mathbf{T}_2$ is $\mathbf{T}_2 \approx \mathcal{C}\, \sigma^2_{\Delta\omega}$ [4]. Thus

$$\boxed{\mathrm{EMSE}(\hat{\mathbf{f}}, \hat{\omega}) \simeq \mathcal{C}\, \sigma^2_{\Delta\omega}.} \quad (29)$$

That is, the EMSE is approximately proportional to the CFO estimation error variance, with the proportionality factor being *independent* of the training sequence. Thus, training sequences that are optimal for CFO estimation, i.e. minimize $\sigma^2_{\Delta w}$, see, e.g., [6]–[8], seem also very good candidates for *joint* channel and CFO estimation.

5. SIMULATION RESULTS

We present simulation results for channel order $L = 3$ and channel coefficients $\mathbf{h}=[0.001 - \jmath 0.0311, -0.0066 + \jmath 0.0825, -0.9451 + \jmath 0.3051, -0.0144 - \jmath 0.0757]^T$, equalizer order $M = 8$, delay $d = 6$, packet length $N = 300$ and TS length $N_{\mathrm{tr}} = 30$. The data symbols are i.i.d. BPSK. The training symbols, which are also i.i.d. BPSK, have been placed close to the middle of the transmitted packet, i.e., $\xi = \frac{\mathcal{C}_2}{2}$. The binary sequence we use corresponds to the hexadecimal number $198153E6$ (we have observed, through exhaustive search, that this sequence has "good" performance for both CFO and joint channel and CFO estimation).

In Fig. 1, we present the time-averages of the three EMSE terms $\mathbf{T}_1$, $\mathbf{T}_2$ and $\mathbf{T}_3$ in (24), (27) and (28), respectively, and their sum, i.e., the approximate EMSE. We observe that $\mathbf{T}_2$ is very close to the approximate EMSE, while terms $\mathbf{T}_1$ and $\mathbf{T}_3$ are much smaller.

6. CONCLUSION

We considered the impact of the channel and CFO estimation errors on the performance of single-carrier systems with MMSE equalizers. We uncovered that, in many cases of high practical importance, the imperfectly canceled CFO is the main cause of the performance degradation. In these cases, the EMSE is approximately proportional to the CFO estimation error variance, with the proportionality coefficient being independent of the TS, implying that optimal TS design

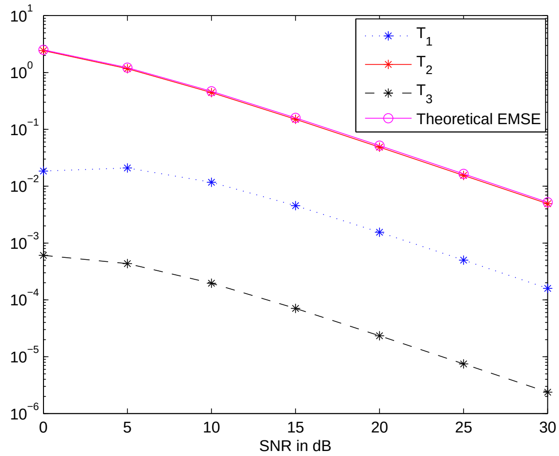

Fig. 1. Final expressions for terms $\mathbf{T}_1$, $\mathbf{T}_2$ and $\mathbf{T}_3$ in (24), (27) and (28) and their sum.

for CFO estimation is also highly relevant for *joint* CFO and channel estimation.

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