Lyapunov Stability of Linear Predictor Feedback for Distributed Input Delays

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Abstract—Compensation of distributed delays in MIMO, LTI systems is achieved using Arstein’s reduction method. In this technical note, we construct a Lyapunov functional for the resulting closed-loop system and establish exponential stability. The key element in our work is the introduction of an infinite-dimensional forwarding-backstepping transformation of the infinite-dimensional actuator states. We illustrate the construction of the Lyapunov functional with a detailed example of a single-input system, in which the input is entering through two individual channels with different delays. Finally, we develop an observer equivalent to the predictor feedback design, for the case of distributed sensor delays and prove exponential convergence of the estimation error.

Index Terms—Delays, distributed parameter systems, linear systems.

I. INTRODUCTION

Delay compensation for linear [1], [2], [8], [10], [11], [15], [16] and nonlinear [5], [6], [12] systems is achieved using predictor-based techniques [4], [13]. In [1] (see also [11]) the following system is considered:

\[ \dot{X}(t) = AX(t) + \int_0^D B(\sigma)U(t-\sigma) d\sigma \] (1)

where \( X(t) \in \mathbb{R}^n, U(t) \in \mathbb{R}^m \) and \( D > 0 \). Such systems can appear in population dynamics [1], in networked control systems [14], or in liquid mono-propellant rocket motors (see [16] and the references therein). For this system and under the controllability condition of the pair

\[ (A, \int_0^D e^{-\lambda \sigma} B(\sigma) d\sigma) \] (2)

the following controller was developed which achieves asymptotic stability for any \( D > 0 \):

\[ U(t) = Ke^{AD}(X(t) + \int_0^D \int_0^t e^{A(t-s-\sigma)} B(\sigma) U(s)d\sigma ds) \] (3)

where the control gain \( K \) may be designed by a LQR/Riccati approach, pole placement, or some other method that makes \( A + \int_0^D e^{-\lambda \sigma} B(\sigma) d\sigma K \) Hurwitz. The above approach (although neither explicitly stated in [1] nor in [11]) works in the case of multi-input systems with different delays in each individual input channel. To see this, one can consider for example the case where \( B(\sigma) = [B_1(\sigma) \quad B_2(\sigma)] \), with

\[ B_0(\sigma) = \begin{cases} B_1(\sigma), & \text{if } \sigma \leq D_1 \\ 0, & \text{if } \sigma > D_1 \end{cases} \] (4)

In this case system (1) becomes

\[ \dot{X}(t) = AX(t) + \sum_{i=1}^2 \int_0^{D_i} B_i(\sigma)U_i(t-\sigma) d\sigma \] (5)

where \( D_2 = D > D_1 \).

However, a Lyapunov functional for the closed-loop systems (1) and (3) is not available. The benefits of constructing a Lyapunov functional for predictor feedback controllers was highlighted in [7]. Having a Lyapunov functional available, one can derive an inverse-optimal controller, prove robustness of the predictor feedback to a small mismatch in the actuator delay or study the disturbance attenuation properties of the closed-loop system. In [7], the following single-input system is considered:

\[ \dot{X}(t) = AX(t) + BU(t-D) \] (6)

together with the predictor-based controller

\[ U(t) = Ke^{AD}X(t) + K \int_0^t e^{A(t-\sigma)} BU(\sigma)d\sigma \] (7)

from [1], [10], [11]. Inverse optimality and robustness to delay mismatch is then proved for the closed-loop systems (6) and (7), using a Lyapunov functional. In addition, a time-varying Lyapunov functional is constructed in [9] for the case of system (6) but when the delay is a function of time i.e., \( D = d(t) \). The Lyapunov functionals in [7], [9] are constructed based on the backstepping method for PDEs [8].

Yet, the backstepping method is not applicable neither in the case of single-input systems with distributed input delay nor in the case of multi-input systems with different delays in each individual input channel. This is since the system that is comprised of the finite-dimensional state \( X(t) \) and the infinite-dimensional actuator states \( U(t + x - D), x \in [0, D] \), are not in the strict-feedback form. In this technical note, a novel forwarding-backstepping transformation of the infinite-dimensional actuator states is introduced to transform the system to an exponentially stable system, whose stability properties can be studied using a quadratic Lyapunov functional. By explicitly finding the inverse transformations, exponential stability of the system in the original variables is established.

We start in Section II with an introduction of the predictor feedback under multiple distributed input delays, and present a transformation of the finite-dimensional state \( X(t) \) from [1], [11]. Then, we establish exponential stability of the closed-loop system using our novel infinite-dimensional transformations of the actuator states. In Section III we develop a dual of the predictor-based controller and design an infinite-dimensional observer which compensates the sensor delays. Finally, in Section IV we present an example that is worked out in detail to demonstrate the construction of the Lyapunov functional.

II. PREDICTOR FEEDBACK FOR DISTRIBUTED INPUT DELAYS

We consider the system

\[ \dot{X}(t) = AX(t) + \sum_{i=1}^2 \int_0^{D_i} B_i(\sigma)U_i(t-\sigma) d\sigma \] (8)

where \( X(t) \in \mathbb{R}^n, U_1(t), U_2(t) \in \mathbb{R} \) and \( D_1, D_2 > 0 \). For notational simplicity we consider a two-input case. The same analysis can be carried out for an arbitrary number of inputs with different delays in each individual input channel. For this system, controller
(3) achieves asymptotic stability for any $D_1, D_2 > 0$ under the controllability condition of the pair $(A, [B_{D_1}, B_{D_2}])$, where $B_i = \int_0^{D_i} e^{-\sigma A} B_i(\sigma) d\sigma$, $i = 1, 2$. The predictor feedback (3) can be written as

$$
\begin{align*}
U(t) &= 
\begin{bmatrix}
U_1(t) \\
U_2(t)
\end{bmatrix} \\
&= 
\begin{bmatrix}
K_1 e^{AD_1} \\
K_2 e^{AD_2}
\end{bmatrix} Z(t) \\
Z(t) &= X(t) + \sum_{i=1}^2 \int_0^{D_i} \int_0^{D_i} e^{A(D_i - y - \sigma)} B_i(\sigma) d\sigma \times U_i(t + y - D_i) dy,
\end{align*}
$$

(9)

and

$$
\begin{align*}
\Omega(t) &\leq \mu \Omega(0) e^{-\rho t} \\
\Omega(t) &= |X(t)|^2 + \sum_{i=1}^2 \int_0^{D_i} U_i^2(t - \theta) d\theta,
\end{align*}
$$

(11)

Moreover, if the initial conditions $u_i(\cdot,0)$, $i = 1, 2$ are compatible with the control law (9) and belong to $H^1(0, D_i)$, $i = 1, 2$, then $(X(t), u_1(t, x), u_2(t, x)) \in C^1([0, \infty), \mathbb{R}^n \times L^2(0, D_1) \times L^2(0, D_2)) \cap C([0, \infty), \mathbb{R}^n \times H^1(0, D_1) \times H^1(0, D_2))$ is the classical solution of the closed-loop system.

Remark 2.1: Before we start the proof of Theorem 1 we give some insight into the main challenges that one faces by considering the problem of constructing a Lyapunov functional for system (5) instead of system (6) that was considered in [7]. System (6) is an ODE-PDE cascade in the strict feed-forward form. To see this we re-write (6) as $\dot{X}(t) = AX(t) + Bu(t)$, where $u(t)$ satisfies $\partial_t u(x, t) = \partial_2 u(t, x)$, and $u_1(t) = U(t)$. One can now observe that the finite-dimensional state $X(t)$ and the infinite-dimensional state $u(x, t), x \in [0, D]$ are in the strict-feedback form since $u(x, t)$ affects the $X(t)$ block only through $x = 0$. On the other hand in (5) it is clear that $u(x, t)$ affects the $X(t)$ block through all of the spatial domain $x \in [0, D]$ and hence the overall system is not in the strict-feedback form. For this reason, a spatially casual backstepping transformation of the infinite-dimensional actuator state which employs a Volterra integral, as the one in [7] and [8], does not apply.

Proof: We first rewrite the plant (8) as

$$
\dot{X}(t) = AX(t) + \sum_{i=1}^2 \int_0^{D_i} \int_0^{D_i} e^{-\sigma A} B_i(\sigma) d\sigma \times U_i(t + y - D_i) dy,
$$

(13)

and

$$
\begin{align*}
\partial_t u_1(x, t) &= \partial_2 u_1(t, x) \\
u_1(t) &= U_1(t) \\
\partial_t u_2(z, t) &= \partial_2 u_2(t, z) \\
w_2(t) &= U_2(t)
\end{align*}
$$

(14)-(17)

where $x \in [0, D_1]$ and $z \in [0, D_2]$. Note that $u_1(x, t) = U_1(t + x - D_1)$ and $u_2(z, t) = U_2(t + z - D_2)$. Note also that the transformation (10) can be written as

$$
Z(t) = X(t) + \sum_{i=1}^2 \int_0^{D_i} \int_0^{D_i} e^{A(D_i - y - \sigma)} B_i(\sigma) d\sigma \times U_i(y, t) dy
$$

(18)

Consider the transformations of the infinite-dimensional actuator states $u_1(x, t)$ and $u_2(z, t)$

$$
\begin{align*}
w_1(x, t) &= u_1(x, t) - \gamma_1(x) X(t) \\
w_2(z, t) &= u_2(z, t) - \gamma_2(z) X(t)
\end{align*}
$$

(19)

where the kernels $\gamma_1(\cdot), p_1(\cdot), k_1(\cdot)$ and $q_i(\cdot)$ for $i = 1, 2$ are to be specified. Using transformations (18)–(20) and relations (9)–(10) we transform the plant (13–17) into the “target system”

$$
\dot{Z}(t) = A + B_1 K_1 e^{AD_1} + B_2 K_2 e^{AD_2} Z(t)
$$

(21)

$$
\begin{align*}
\partial_t w_1(x, t) &= \partial_2 w_1(t, x) - q_1(x, D_1) K_1 e^{AD_1} Z(t) \\
w_1(D_1, t) &= 0 \\
\partial_t w_2(z, t) &= \partial_2 w_2(t, z) - q_2(z, D_1) K_1 e^{AD_1} Z(t) \\
w_2(D_2, t) &= 0
\end{align*}
$$

(22)-(25)

We first prove (21). Differentiating with respect to time (18) and using (13–14), (16), we obtain

$$
\dot{Z}(t) = AX(t) + \sum_{i=1}^2 \int_0^{D_i} \int_0^{D_i} e^{A(D_i - y - \sigma)} B_i(\sigma) d\sigma \times \partial_2 u_i(y, t) dy
$$

(26)

where $x \in [0, D_1]$ and $z \in [0, D_2]$. Note that $u_1(x, t) = U_1(t + x - D_1)$ and $u_2(z, t) = U_2(t + z - D_2)$. Note also that the transformation (10) can be written as

$$
Z(t) = X(t) + \sum_{i=1}^2 \int_0^{D_i} \int_0^{D_i} e^{A(D_i - y - \sigma)} B_i(\sigma) d\sigma \times U_i(y, t) dy
$$

(18)

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\end{align*}
$$

(19)

where the kernels $\gamma_1(\cdot), p_1(\cdot), k_1(\cdot)$ and $q_i(\cdot)$ for $i = 1, 2$ are to be specified. Using transformations (18)–(20) and relations (9)–(10) we transform the plant (13–17) into the “target system”

$$
\dot{Z}(t) = A + B_1 K_1 e^{AD_1} + B_2 K_2 e^{AD_2} Z(t)
$$

(21)

$$
\begin{align*}
\partial_t w_1(x, t) &= \partial_2 w_1(t, x) - q_1(x, D_1) K_1 e^{AD_1} Z(t) \\
w_1(D_1, t) &= 0 \\
\partial_t w_2(z, t) &= \partial_2 w_2(t, z) - q_2(z, D_1) K_1 e^{AD_1} Z(t) \\
w_2(D_2, t) &= 0
\end{align*}
$$

(22)-(25)

We first prove (21). Differentiating with respect to time (18) and using (13–14), (16), we obtain

$$
\dot{Z}(t) = AX(t) + \sum_{i=1}^2 \left( \int_0^{D_i} B_i(D_i - y) u_i(y, t) dy + \int_0^{D_i} \int_0^{D_i} e^{A(D_i - y - \sigma)} B_i(\sigma) d\sigma \times \partial_2 u_i(y, t) dy \right)
$$

(26)
Using (9) we get (22)–(23). Similarly, one obtains the kernels of (20) which can be derived from (26)–(29) by changing the index 1 into 2 and the spatial variable $x$ into $z$. The next step is deriving the inverse transformations of (19)–(20). We postulate the inverse transformation of (19) in the form

$$ u_1(x, t) = w_1(x, t) + \delta_1(x)Z(t) - \int_x^{D_1} m_1(x, y)w_1(y, t)dy. \quad (30) $$

Taking the time and the spatial derivatives of the above transformation, using integrations by parts together with (21)–(23) and the fact that $q_1(x, D_2) = K_1e^{A_2z}B_{D_2}$, after subtracting the resulting expressions for the time and spatial derivatives, we conclude that relations (14)–(15) hold (by taking into account also (9)) if

$$ \delta_1(x) = K_1e^{A_2x}e^{A_2z(x-D_2)} + K_1e^{A_2y}B_{D_2}K_2e^{A_2z}e^{A_2y}dy - \int_x^{D_1} g_1(y - r)K_1e^{A_2z}dy. \quad (31) $$

To establish that (30) is indeed the inverse transformation of (19) one has to uniquely determine $g_1(x - y)$. We find next an explicit expression for $g_1(x - y)$. To do so, we substitute $Z(t)$ and $w_1(x, t)$ from (18) and (19) respectively, into (30). Matching the terms for $X(t)$, $u_1(y, t)$, and $w_2(y, t)$ and after some algebraic manipulations we conclude that $g_1(x - y)$ must satisfy for all $x \in [0, D_1]$

$$ 0 = -K_1e^{A_2x} + \delta_1(x) + \int_x^{D_1} g_1(x - y)K_1e^{A_2y}dy \quad (31) $$

Using (34) we can simplify $\delta_1(x)$. After some algebra we rewrite (31) as

$$ \delta_1(x) = K_1e^{A_2z}B_{D_2}K_2e^{A_2z}, \quad (35) $$

Finally, after some algebra one can verify that indeed relations (34)–(35) satisfy condition (31). Similarly, the inverse transformation of (20) is given by

$$ w_2(z, t) = w_2(z, t) + \delta_2(z)Z(t) - \int_z^{D_2} m_2(z, y)w_2(y, t)dy. \quad (36) $$

and $B_{D_2}e^{A_2z} = B_{D_2}e^{A_2z}B_{D_2}K_2e^{A_2z}B_{D_2}, \quad m_2(z, y) = \frac{g_2(z - y)}{K_2e^{A_2z}} = K_2e^{A_2z}B_{D_2}K_2e^{A_2z}, \quad \delta_2(z) = K_2e^{A_2z}. \quad (37) $$

Using (18) together with (30) and (36) we get $X(t)$ as a function of $w_1(y, t)$ and $w_2(y, t)$. We have

$$ X(t) = \left( I - \sum_{i=1}^{2} \int_0^{D_1} e^{A_2z}B_i(\sigma)d\sigma \right)Z(t) - \sum_{i=1}^{2} \int_0^{D_1} e^{A_2z}B_i(\sigma)d\sigma \left( w_1(y, t) + \int_y^{D_2} m_1(y, r)w_2(r, t)dr \right)dy. \quad (37) $$

Consider now the Lyapunov functional for the target system (21)–(25)

$$ V(t) = Z(t)^T PZ(t) + \alpha \int_0^{D_1} (1 + x)w_1^2(x, t)dx + \beta \int_0^{D_2} (1 + z)w_2^2(z, t)dz \quad (38) $$

where the positive parameters $\alpha$, $\beta$ are to be chosen later and $P = P^T > 0$ is the solution to the following Lyapunov equation:

$$ (A + B_{D_1}K_2e^{A_2z} + B_{D_2}K_2e^{A_2z})^TP + P (A + B_{D_1}K_2e^{A_2z} + B_{D_2}K_2e^{A_2z}) = -Q \quad (39) $$

for some $Q = Q^T > 0$, $K_1$, and $K_2$. For the time derivative of $V(t)$ along (21)–(25) we obtain $\dot{V}(t) \leq -\lambda_{\text{min}}(Q)Z(t)^TP - \alpha w_1^2(0, t) - \beta w_2^2(0, t) - \int_0^{D_1} \left( \frac{\delta_1}{\sqrt{2}} \right)^2 (1 + x)K_1e^{A_2z}B_{D_2}K_2e^{A_2z}\sqrt{Z}(t) + \beta w_2^2(0, t) - \beta \int_0^{D_2} w_2^2(z, t)dz - \int_0^{D_2} 2(\sqrt{2})w_2(z, t)(1 + z)K_2e^{A_2z}B_{D_2}K_2e^{A_2z}\sqrt{Z}(t), \quad (39) $$

where $\lambda_{\text{min}}(Q)$ is the smallest eigenvalue of $Q$ in (39). Applying Young’s inequality we get $\dot{V}(t) \leq -\lambda_{\text{min}}(Q) - \alpha \gamma - \beta \delta$.
applying Young and Cauchy-Schwartz’s inequalities we get

\[ |Z(t)|^2 \leq (\alpha/2) \int_0^{T_d} u_i^2(t,x,t) dt + (\beta/2) \int_0^{T_d} u_2^2(z,t) dt, \]

where \( \gamma = 2D_1(1 + D_1)K \left( \alpha D_1 \right) B_{D_2}K_2 \alpha D_2 \) and \( \delta = 2D_2(1 + D_2)K_2 \alpha D_2 \). Hence by choosing \( \alpha = \lambda_{\min}(Q) / \gamma \) and \( \beta = \lambda_{\min}(Q) / 4\delta \), and by setting \( \rho = \min \{ \lambda_{\min}(Q) / 2^{\lambda_{\max}(P), \alpha / (2 + D_1), \beta / (2 + D_2) \} \}

\[ \lambda_{\max}(P) \] is the largest eigenvalue of \( P \) in (39) we get \( \hat{V}(t) \leq -\lambda(\hat{V}(t)) \). Using the comparison principle we get \( \hat{V}(t) \leq V(0)e^{-\rho t} \). To prove stability in the original variables \( X(t) \)

and \( u_1(t,x,t) \) it is sufficient to show that

\[ M(\Omega(t)) \leq V(t) \leq M(\Omega(t)) \]

for some positive \( M \) and \( \bar{M} \), since then \( \Omega(t) \leq (\bar{M}/M)(\Omega(t))e^{-\rho t} \), and Theorem 1 is proved with \( \mu = \bar{M}/M \). Using (18), and applying Young and Cauchy-Schwartz’s inequalities we get

\[ |Z(t)|^2 \leq m_1||X(t)||^2 + \int_0^{T_d} u_i^2(t,x,t) dt + \int_0^{T_d} u_2^2(z,t) dt, \]

where \( m_1 = 3 \max \{ 1, \max_{i=1,2} \left( \int_0^{T_d} e^{|x(t)|^2} B_i(t)|x(t)|^2 \right) \} \). Moreover, using (19)–(20) together with Young and Cauchy-Schwartz’s inequalities we get

\[ \int_0^{T_d} u_i(t,x,t) dt + \int_0^{T_d} u_2(t,z,t) dt, i = 1, 2, \]

where \( m_i = 4D_1 \max \{ 1, f_{1i}, f_{2i} \}, f_{1i} = \sup_{y \in [0, D_1]} \gamma_i(y), f_{2i} = D_2 \sup_{y \in [0, D_2]} \gamma_i(y), |x(y)|^2 \}

\( f_{3i} = D_3 \sup_{y \in [0, D_3]} \gamma_i(y), f_{4i} = i, j = 1, 2, i \neq j \). With \( \bar{M} = \max \{ \lambda_{\max}(P), \alpha (1 + D_1), \beta (1 + D_2) \} \max \{ m_1, m_2, m_3 \}

the upper bound in (40) is proved. Similarly using (30), (36), (37) we get \( \bar{M} = \min \{ \lambda_{\max}(P), \alpha \beta / \max \{ b_1, b_2, b_3 \} \}

where \( \bar{M} = \min \{ 1, 2, \}

(42)

which can be written equivalently as

\[ \dot{\hat{X}}(t) = AX(t) + BU(t), \]

\[ Y_i(t) = \int_0^{T_d} C_1(t, \sigma) X(t - \sigma) d\sigma \]

\[ Y_2(t) = \int_0^{T_d} C_2(t, \sigma) X(t - \sigma) d\sigma \]

and by noting that \( \gamma_1(x) \) and \( \gamma_2(z) \) in (52)–(53) are the solutions of the boundary value problems \( \gamma_1(x) = \gamma_1(x)A - C_1(0) \) together with \( \gamma_1(D_1) = 0 \) and \( \gamma_2(z) = \gamma_2(z)A - C_2(0) \) together with \( \gamma_2(D_2) = 0 \), we get

\[ \tilde{X}(t) = (A - L_1C_1 - L_2C_2) \tilde{X}(t) \]

\[ \tilde{Y}_1(t) = \tilde{Y}_1(0,t) \]

\[ \tilde{Y}_2(t) = \tilde{Y}_2(0,t) \]

Next we state a new observer that compensates the sensor delays and prove exponential convergence of the resulting observer system.

\[ \text{Theorem 2: Consider the system } (41) \text{ and let the pair } (A, \left[ \begin{array}{c} C_{D_1} \\ C_{D_2} \end{array} \right] ) \text{ be observable, where } \]

\[ C_{D_1} = \int_0^{T_d} C_1(t, \sigma) e^{-\beta \sigma} d\sigma, \quad i = 1, 2. \]

Define the observer

\[ \hat{X}(t) = AX(t) + BU(t) \]

\[ \hat{Y}_1(t) = \hat{Y}_1(0,t) \]

\[ \hat{Y}_2(t) = \hat{Y}_2(0,t) \]

where \( \hat{X}(t) \) and \( \hat{Y}_i(t) \) are the estimates of \( X(t) \) and \( Y_i(t) \), respectively. The observer error system is given by

\[ \left[ \begin{array}{c} \dot{\tilde{X}}(t) \\ \dot{\tilde{Y}}_1(t) \\ \dot{\tilde{Y}}_2(t) \end{array} \right] = \left[ \begin{array}{c} (A - L_1C_1 - L_2C_2) \tilde{X}(t) \\ L_1 \tilde{Y}_1(t) - \hat{Y}_1(t) \\ L_2 \tilde{Y}_2(t) - \hat{Y}_2(t) \end{array} \right] + \left[ \begin{array}{c} 0 \\ \tilde{Y}_1(0,t) \\ \tilde{Y}_2(0,t) \end{array} \right] \]

and by noting that \( \gamma_1(x) \) and \( \gamma_2(z) \) in (52)–(53) are the solutions of the boundary value problems \( \gamma_1(x) = \gamma_1(x)A - C_1(0) \) together with \( \gamma_1(D_1) = 0 \) and \( \gamma_2(z) = \gamma_2(z)A - C_2(0) \) together with \( \gamma_2(D_2) = 0 \), we get

\[ \tilde{X}(t) = (A - L_1C_1 - L_2C_2) \tilde{X}(t) \]

\[ \tilde{Y}_1(t) = \tilde{Y}_1(0,t) \]

\[ \tilde{Y}_2(t) = \tilde{Y}_2(0,t) \]
\[ \partial_t \dot{\xi}(z,t) = \partial_z \dot{\xi}(z,t) \]  

(61)

\[ \dot{\xi}(D_2 t, t) = 0. \]  

(62)

To establish exponential stability of the error system we use the following Lyapunov functional

\[ V(t) = \dot{X}^T(t) P \dot{X}(t) + \alpha \int_0^{D_1} (1 + x) \dot{\xi}_2^2(x,t) dx + \beta \int_0^{D_2} (1 + z) \dot{\xi}_3^2(z,t) dx \]  

(63)

where the positive parameters \( \alpha, \beta \) are to be chosen later and \( P = P^T > 0 \) satisfies the Lyapunov equation

\[ (A - L_1C_{D_1} - L_2C_{D_2})^T P + P(A - L_1C_{D_1} - L_2C_{D_2}) = -Q, \]

for some \( Q = Q^T > 0, L_1 \) and \( L_2 \). Taking the time derivative of \( V(t) \), using integration by parts in the integrals in (63) and relations (58)–(62) we get

\[ \dot{V}(t) \leq -\lambda_{\min}(Q) \| \dot{X}(t) \|^2 - \alpha \int_0^{D_1} \dot{\xi}_2^2(x,t) dx - \alpha \int_0^{D_2} \dot{\xi}_3^2(z,t) dz - \beta \dot{\xi}_3^2(z_0,t) + \left( \sqrt{\lambda_{\min}(Q)} \| \dot{X}(t) \| / \sqrt{2} \right) \left( 2\sqrt{\| P \| \| L_1 \| \sqrt{\lambda_{\min}(Q)} \right) \left( |c(t,0)| + \left( \sqrt{\lambda_{\min}(Q)} \| \dot{X}(t) \| / \sqrt{2} \right) \left( 2\sqrt{\| P \| \| L_2 \| \sqrt{\lambda_{\min}(Q)} \dot{\xi}_3(z_0,t) \right) \right. \]

(64)

Relation (30) together with the facts \( e^{A_{\Delta} t} \alpha \) and \( \lambda_{\max}(P) \) yield the inverse transformation of (68) as

\[ u(x,t) = w(x,t) + K e^{-A_D t} Z(t) - K e^{-A_D t} \int_x^D e^{A_D (t-s)} B u(s,y) ds dy \]  

(73)

Now one can use the Lyapunov functional (38) to establish exponential stability of the closed-loop system (64)–(66), (69). Instead, we will use the following Lyapunov functional

\[ V(t) = \dot{X}^T(t) P \dot{X}(t) + \alpha \int_0^{D_1} (1 + x) u^2(x,t) dx \]  

(75)

where \( \alpha \) is an arbitrary positive constant. Observe here that the above Lyapunov functional depends on \( X(t) \) directly and through \( u(x,t) \), while the Lyapunov functional defined in (38) depends on \( X(t) \) through \( Z(t) \) and \( u(x,t) \). Hence, in order to prove exponential stability of the closed-loop system (64)–(66), (69), using (75), we have to derive a relation for the \( (X,u) \) cascade. To see this one has to solve (74) for \( Z(t) \) (assuming that the matrix that multiplies \( Z(t) \) is invertible) and then plugging the resulting expression into (73) and (69). Then, having on the right hand side of (73) and (69) only \( X(t) \) and \( u(x,t) \) one can find \( u(t) \) and \( u(D,t) \) only as a function of \( X(t) \) and \( u(x,t) \). Plugging into (64) \( u(t) \) and \( u(D,t) \) we get the \( (X,u) \) cascade. One can show using (75) that this cascade is exponentially stable.
In this work, we prove exponential stability of predictor feedback in multi-input systems with distributed input delays. We reach our result by constructing a Lyapunov functional for the closed-loop system, based on novel transformations of the actuator states. Furthermore, we design an observer for multi-output systems with distributed sensor delays and prove exponential convergence of the estimation error. Finally, a detailed example is presented that illustrates the construction of the Lyapunov functional.

REFERENCES


Dissipativity-Based Switching Adaptive Control

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Abstract—This technical note introduces a new dissipativity-based switching adaptive control strategy for uncertain systems. In this approach, the parametric uncertainties in systems are not considered on continua but studied on finite discrete sets. Differently to multiple model supervisory control, dissipativity-based switching adaptive control needs no estimation errors for each switching decision. The switching logic is designed based on the relationship between dissipativity/passivity and adaptive systems, such that in the process of switching control, a transient boundary can be guaranteed by appropriately switching the parameter estimates. This makes it possible to apply the strategy for nonlinear systems modeled in local regions in the state space. We also discuss the implementation of the new idea to general dissipative systems and feedback passive systems. An example with simulation is employed to show the effectiveness of the approach.

Index Terms—Convergence, dissipativity/passivity, switching adaptive control, transient boundary, uncertain nonlinear systems.

I. INTRODUCTION

Transient boundedness is important for both theoretical systems synthesis and practical implementation of the controllers in applications. For instance, in nonlinear systems identification and modeling, because of the inherent nonlinear property, it is usually hard to completely identify the uncertainties. So, the models of nonlinear systems we often use for control design are locally valid along the experienced trajectories in state space and sometimes depend on the operating situation and environments [27]. If the model is only valid in the region $\Omega_c$ in state space, the controlled state trajectory $\varphi$ should not leave $\Omega_c$. More examples

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