Simultaneous Compensation of Actuation and Communication Delays for Heterogeneous Platoons via Predictor-Feedback CACC with Integral Action

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***Abstract*—We construct a predictor-feedback cooperative adaptive cruise control (CACC) design with integral action, which achieves simultaneous compensation of long, actuation and communication delays, for platoons of heterogeneous vehicles whose dynamics are described by a third-order linear system with input delay. The key ingredients in our design are an underlying predictor-feedback law that achieves actuation delay compensation and an integral term of the difference between the delayed (by an amount equal to the respective communication delay) and current speed of the preceding vehicle. The latter, essentially, creates a virtual spacing variable, which can be regulated utilizing only delayed position and speed measurements from the preceding vehicle. We establish individual vehicle stability, string stability, and regulation for vehicular platoons, under the control design developed. The proofs rely on combining an input-output approach (in the frequency domain), with derivation of explicit solutions for the closed-loop systems, and they are enabled by the actuation and communication delays-compensating property of the design. We demonstrate numerically the control and model parameters' conditions of string stability, while we also present simulation results, in realistic scenarios, including a scenario in which the leading vehicle's trajectory is obtained from NGSIM data. All case studies confirm the effectiveness of the design developed.**

***Index Terms*—Delay compensation, string stability of vehicular platoons, cooperative adaptive cruise control (CACC), predictor feedback, actuation and communication delays.**

I. Introduction

A. Motivation

STRING stability is a crucial requirement and serves as an indicator of the safety and efficiency properties of platoons consisting of vehicles equipped with Adaptive Cruise Control (ACC) and CACC capabilities, see, for example, [11], [22], [24]. This property is imperiled when delays affect actuation, sensing, or communication of vehicular systems, see, for example, [4], [6], [13], [20], [23], [30], [31], [36], [37]. In particular, communication delay, stemming from vehicle-to-vehicle (V2V) communication, imposes a significant challenge to string stability, particularly when both actuation and communication delays coexist and they are large [10], [18], [22], [26], [33], [36].

The research in this paper was supported by the Hellenic Foundation for Research and Innovation (H.F.R.I.) under the "2nd Call for H.F.R.I. Research Projects to Support Faculty Members & Researchers" (Project Number: 3537/ORAMA).

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B. Literature

For this reason, compensation of such delays becomes an essential mechanism that could be integrated with nominal, ACC/CACC laws. This integration may lead to a substantial enhancement in string stability properties of vehicular platoons. This is already evident in works that address small actuation delays only [14], [30], or small communication delays only [1], [10], [23], [25], or both [5], [13], [18], [22], [33], [35]. To address larger actuation or communication delays a predictor-based approach is required. Predictor-based control designs addressing long actuation and communication delays can be found in [3], [4], [6], [12], [19], [20], [28], [31], [34] and [32], respectively; while [36] presents a predictor-based design to address both long actuation and communication delays. Our work is viewed as complementary and different to [36], in the sense of a) developing a new, less complex predictor-feedback CACC design with integral action, b) achieving simultaneous compensation of large actuation/communication delays, c) developing a constructive and systematic approach for establishing individual vehicles' stability, string stability, and regulation of the closed-loop systems, and d) validating our design with real traffic data. For the reader's benefit, we present Table I, which illustrates the distinctions between the present work and related, existing works.

TABLE I
VARIOUS TYPES OF DELAYS ADDRESSED IN LITERATURE

Delays Types \ References	[3], [4], [6], [12], [19], [20], [28], [31], [34]	[32]	[36] and current paper	[1], [5], [10], [13], [14], [18], [22], [23], [25], [30], [33], [35]
Small actuation and/or communication delays				✓
Large actuation delay *only*	✓			
Large communication delay *only*		✓		
Simultaneous large actuation and communication delays			✓	

In the present paper, we build upon the predictor-feedback CACC law from [4], which is constructed to compensate actuation delay only. While in [26] it is established that string stability of the CACC law from [4] is robust to small communication delay, a predictor-feedback CACC design addressing simultaneously, long actuation and communication delays is not available. The main reason for this unavailability is the fact that exact predictor states (over a prediction horizon equal to the actuation delay) cannot be constructed anymore in the presence of communication delays. Nevertheless, as we establish here, to achieve string stability it is not necessarily required to construct exact predictor states, but to rather cancel the effect of communication delay by aiming at regulation of spacing and speed of the ego vehicle, essentially, to the past (rather than the current) spacing and speed of the preceding vehicle.

C. Contributions

Towards this end, we construct a linear, predictor-feedback CACC law augmented with an integral term of the difference between the preceding vehicle's, delayed, by an amount equal to the respective communication delay, speed and its current speed. We consider platoons of vehicles with heterogeneous dynamics described by a third-order linear system with actuation delay. The control design developed achieves $\mathcal{L}_2$ string stability with respect to speed/acceleration errors propagation (and with respect to spacing errors propagation as well, in the particular case of homogeneous vehicles). String stability is achieved relying on the following two mechanisms embedded in the control law developed–an underlying predictor-feedback CACC design that aims at actuation delay compensation and the integral term of the difference between the delayed and current speed of the preceding vehicle (which, in fact, may be viewed as a type of spacing variable). The latter, essentially, modifies the objective of the original control law to aiming at regulating the spacing (and speed) of the ego vehicle accounting for the delayed, rather than the current, position and speed of the preceding vehicle. This, in a way, aligns the regulation objectives of the controller with the available information for the preceding vehicle's state at the current time, which is beneficial for string stability. Furthermore, the control design achieves stability of individual vehicles (which is a prerequisite for string stability) and zero, steady-state speed and spacing tracking errors, for a constant leader's speed. To achieve zero, steady-state spacing tracking error it is required to reduce the original time-headway by an amount equal to the respective communication delay, which imposes a condition that the desired time-headway is larger than the respective communication delay. This, in fact, is reasonably expected since the controller reacts to past rather than current information of the preceding vehicle's state. Nevertheless, the values of actuation and communication delays themselves are not restricted.

The proof of string stability relies on an input-output approach, deriving the respective transfer functions between the speed of the ego and the preceding vehicle, together with deriving explicit conditions on control/model parameters and time-headway. The proofs of individual vehicle stability and regulation rely on deriving explicit solutions of the closed-loop systems, capitalizing on the ability of the control design developed to achieve actuation and communication delays compensation. The analytical string stability conditions are also illustrated numerically. Furthermore, we present consistent simulation results of a platoon of ten vehicles, for the practical scenario in which a vehicle cuts in the platoon (described by considering initial condition deviations from equilibrium) and it subsequently performs an acceleration/deceleration maneuver. As it is shown, the performance of the platoon is considerably improved as compared with [26], since only actuation but not communication delay compensation is achieved in [26], and thus, for large values of communication delays the design from [26] cannot guarantee string stability. In addition, we validate the design considering a scenario in which the leading vehicle's acceleration is obtained from the NGSIM data.

D. Organization

The outline of the paper is as follows. Section II presents the model of heterogeneous platoons considered and the communication/actuation delays-compensating predictor-feedback design with integral action. In Section III, we state our main result, which is stability, string stability, and regulation under the CACC law developed, whose proof is provided in Appendix A. In Section IV we present numerical experiments for validation of the string stability guarantees. Simulation results are presented in Section V and in Section IV we provide concluding remarks.

II. Predictor-Feedback CACC for Heterogeneous Platoons with Both Actuator and Communication Delays

A. Vehicle Model and Nominal Delay-Free Design

a) Vehicle dynamics: We consider a heterogeneous string of vehicles (see Fig. 1) each one modeled by the following third-order, linear system with actuator delay that describes vehicle dynamics (see, e.g., [1], [31], [32], [33])

$$\dot{s}_i(t) = v_{i-1}(t) - v_i(t), \tag{1}$$

$$\dot{v}_i(t) = a_i(t), \tag{2}$$

$$\dot{a}_i(t) = -\frac{1}{\tau_i}a_i(t) + \frac{1}{\tau_i}u_i(t-D), \tag{3}$$

$i = 1, ..., N$, where $s_i = x_{i-1} - x_i - l$ and x_i is the position of vehicle i and l is its length, v_i is vehicle speed, a_i is vehicle acceleration, τ_i is lag, capturing, engine dynamics, u_i is the individual vehicle's control variable, $D \geq 0$ is input delay, and $t \geq 0$ is time. Note that for the leading vehicle we assume similarly that it has the same type of third-order dynamics as the rest of the vehicles. The difference is that u_l acts as a time-varying, exogenous input rather than as feedback control input. We adopt the convention that $v_0 = v_l$ and $a_0 = a_l$ are the speed and acceleration of the string leader, respectively.

b) Available measurements: For CACC platoons the measurements available to the ego vehicle i are its own spacing s_i, speed v_i, acceleration a_i, and control input u_i as well as the speed of the preceding vehicle v_{i-1}. It is possible to obtain this information through on-board sensors (for a more low level description of the ways of acquiring such measurements in practical applications the reader is referred to, e.g., [7]– [9]). Furthermore, the control input of the preceding

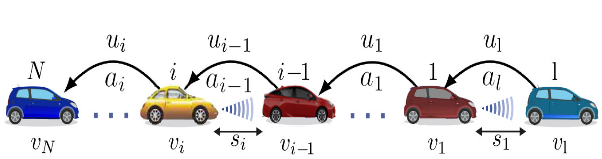

Fig. 1. Platoon of $N+1$ heterogeneous vehicles following each other in a single lane without overtaking. The dynamics of each vehicle $i=1,...,N$ are governed by system (1)–(3). Each vehicle can measure its own speed, the relative speed with the preceding vehicle, and the spacing with respect to the preceding vehicle. The control input and acceleration of each vehicle is communicated to the following vehicle via V2V communication.

vehicle, as well as its acceleration and speed are also available and are denoted by $u_{i-1,\mathrm{m}}$, $a_{i-1,\mathrm{m}}$, and $v_{i-1,\mathrm{m}}$ respectively. These measurements are transmitted from the preceding vehicle, through V2V communication. Due to the presence of communication delay these measurements are modeled by $v_{i-1,\mathrm{m}}(t)=v_{i-1}(t-D_{\mathrm{c},i-1})$, $a_{i-1,\mathrm{m}}(t)=a_{i-1}(t-D_{\mathrm{c},i-1})$ and $u_{i-1,\mathrm{m}}(\theta)=\ u_{i-1}(\theta-\ D_{\mathrm{c},i-1}),\theta\in[t-D,t]$, respectively, where $D_{\mathrm{c},i-1}\geq 0$, $i=1,...,N$, are communication delays[1].

c) Nominal control design: Without input delay, the following control strategy is constructed

$$\begin{aligned}u_i(t)=\tau_i\alpha_i\left(\frac{s_i(t)}{h_i}-v_i(t)\right)+\tau_i b_i(v_{i-1}(t)-v_i(t))\\+\tau_i c_i a_i(t),\end{aligned}\tag{4}$$

where $\alpha_i>0$, $b_i>0$, and $c_i\in\mathbb{R}$ are design parameters, and $h_i>0$ is time-headway.

B. Communication Delay-Compensating Predictor-Feedback Control Design

The predictor-based control laws with communication delay compensation for system (1)–(3) are given by

$$\begin{aligned}u_i(t)=&\frac{\tau_i\alpha_i}{h_i}q_{i,1}(t)-\tau_i(\alpha_i+b_i)q_{i,2}(t)+\tau_i b_i q_{i,3}(t)\\&+\tau_i c_i q_{i,4}(t)+\frac{\tau_i\alpha_i}{h_i}\sigma_i(t),\end{aligned}\tag{5}$$

$$\dot{\sigma}_i(t)=v_{i-1,\mathrm{m}}(t)-v_{i-1}(t),\tag{6}$$

$$\begin{aligned}q_i(t)=&\,\mathrm{e}^{\Gamma_i D}\bar{x}_i(t)+\int_{t-D}^{t}\mathrm{e}^{\Gamma_i(t-\theta)}B_i u_i(\theta)\,d\theta\\&+\int_{t-D}^{t}\mathrm{e}^{\Gamma_i(t-\theta)}B_{1i}u_{i-1,\mathrm{m}}(\theta)\,d\theta,\end{aligned}\tag{7}$$

where

$$q_i=\begin{bmatrix}q_{i,1}\\q_{i,2}\\q_{i,3}\\q_{i,4}\\q_{i,5}\end{bmatrix},\quad \bar{x}_i=\begin{bmatrix}s_i\\v_i\\v_{i-1,\mathrm{m}}\\a_i\\a_{i-1,\mathrm{m}}\end{bmatrix},\tag{8}$$

$$B_i=\begin{bmatrix}0&0&0&\frac{1}{\tau_i}&0\end{bmatrix}^{\mathrm{T}},B_{1i}=\begin{bmatrix}0&0&0&0&\frac{1}{\tau_{i-1}}\end{bmatrix}^{\mathrm{T}},\tag{9}$$

[1]The initial conditions $v_{i-1}(s)=v_{i-1_0}(s)$, $s\in[-D_{\mathrm{c},i-1},0]$, $a_{i-1}(s)=a_{i-1_0}(s)$, $s\in[-D_{\mathrm{c},i-1},0]$ and $u_{i-1}(s)=u_{i-1_0}(s)$, $s\in[-D-D_{\mathrm{c},i-1},0)$ are assumed to be continuous functions.

$$\Gamma_i=\begin{bmatrix}0&-1&1&0&0\\0&0&0&1&0\\0&0&0&0&1\\0&0&0&-\frac{1}{\tau_i}&0\\0&0&0&0&-\frac{1}{\tau_{i-1}}\end{bmatrix}.\tag{10}$$

Implementation of control laws (5) requires measurements of the ego vehicle's spacing s_i, speed v_i, acceleration a_i, and control input u_i as well as the relative speed with the preceding vehicle. Utilizing on-board sensors, this information can be obtained. The preceding vehicle's speed $v_{i-1,\mathrm{m}}$, acceleration $a_{i-1,\mathrm{m}}$, and control input $u_{i-1,\mathrm{m}}$, which are also required, can be obtained through V2V communication that, however, are subject to communication delay. It is important to note that we employ in our control design two different measurements for the preceding vehicle speed, one from on-board sensors v_{i-1} and one from V2V communication $v_{i-1,\mathrm{m}}$. Note that for control implementation the value of the communication delay is not needed, because $v_{i-1,\mathrm{m}}$ can be obtained directly from V2V communication. If $D_{\mathrm{c},i-1}$ is known to vehicle i, then one could, alternatively, employ $v_{i-1,\mathrm{m}}(t)$ via $v_{i-1,\mathrm{m}}(t)=v_{i-1}(t-D_{\mathrm{c},i-1})$.

Control laws (5), in the absence of communication delay, correspond to an exact predictor-feedback CACC design. In the present case, due to communication delay, the states q_i are not exact, D-time units predictor states anymore. Nevertheless, simultaneous compensation of input and communication delays is achieved, as stated in the next section, in which we also include details on the mechanism embedded in our controller that enables input/communication delays compensation.

III. String Stability Despite Actuation and Communication Delays

We start providing the definition of string stability employed. A platoon of vehicles indexed by $i=1,...,N$, following each other within one lane without overtaking, is $\mathcal{L}_2$ string stable with reference to speed errors if the following condition holds

$$\sup_{\omega}|G_i(j\omega)|\leq 1,\qquad i=1,...,N,\tag{11}$$

where $G_i(j\omega)$ denotes the transfer function between the i-th vehicle's speed and the speed of its preceding vehicle $i-1$ (see, e.g., [11], [13]). It should be noted that string stability of a platoon in the heterogeneous case depends on the selection of states used to analyze the propagation of disturbances (upstream in the platoon). Here we study $\mathcal{L}_2$ string stability with respect to speed errors propagation, as this is the most commonly used definitions, see, for example, [13], [36]. Note also that the respective transfer functions, corresponding to acceleration states, are identical to those for speed states (and, in the case of homogeneous platoons, identical to spacing error states as well). We now state our main result.

Theorem 1: Consider a platoon of vehicles with heterogeneous dynamics modeled by (1)–(3), under control laws (5) with (6)–(10). Let the leading vehicle's speed be uniformly bounded and continuous. For any $D\geq 0$, $h_i>0$, the platoon is $\mathcal{L}_2$ string stable with respect to speed errors propagation provided that the following conditions hold: $\frac{1}{\tau_i} - c_i > 0$, $\left(\frac{1}{\tau_i} - c_i\right)(\alpha_i + b_i) - \frac{\alpha_i}{h_i} > 0$, $\left(c_i - \frac{1}{\tau_i}\right)^2 - 2(\alpha_i + b_i) > 0$, and $\frac{2}{h_i}\left(c_i - \frac{1}{\tau_i}\right) + 2b_i + \alpha_i > 0$, $i = 1, ..., N$. Furthermore, all states remain bounded and, for a constant leading vehicle's speed, say v^*, regulation is achieved with $\lim_{t\to+\infty} a_i(t) = 0$, $\lim_{t\to\infty} v_i(t) = v^*$, and $\lim_{t\to+\infty} s_i(t) = h_i v^* - \lim_{t\to+\infty} \sigma_i(t)$, where $\lim_{t\to+\infty} \sigma_i(t) = \sigma_i(0) + \int_{-D_{\mathrm{c},i-1}}^{0} v_{i-1_0}(s) ds - D_{\mathrm{c},i-1} v^*$, $i = 1, ..., N$.

Proof: The proof can be found in Appendix A.

Remark 1: Communication delay is compensated by regulating the speed of the ego vehicle to match the speed of the preceding vehicle, also accounting for the respective communication delay (see Remarks 2 and 3). This regulatory action in the presence of communication delays alters the equilibrium point, resulting in loss of zero, steady-state tracking error, as the controller aims to regulate $s_i + \sigma_i$ (rather than s_i) to $h_i v_i$ (this phenomenon also appears in, e.g., [32]). To address steady-state error when communication delay is known (e.g., as a known, average network delay), we can set $\sigma_i(0) = -\int_{-D_{\mathrm{c},i-1}}^{0} v_{i-1_0}(s) ds$ and $h_i = h_{i,\mathrm{des}} - D_{\mathrm{c},i-1}$ (assuming $h_{i,\mathrm{des}} > D_{\mathrm{c},i-1}$, which is a reasonable requirement given that the controller reacts with $D_{\mathrm{c},i-1}$ delay and that, typically, the values of communication delay are much smaller that the desired headways), which results in a steady-state value for s_i to be $h_{i,\mathrm{des}} v^*$. Note that the choice for $\sigma_i(0)$ can be implemented at $t = 0$ using the past measurements of v_{i-1}, which are available. On the other hand, if $D_{\mathrm{c},i-1}$ is unknown, we can set $\sigma_i(0) = 0$. This results in a steady-state error for s_i of $D_{\mathrm{c},i-1} v^* - \int_{-D_{\mathrm{c},i-1}}^{0} v_{i-1_0}(s) ds$. Nevertheless, it is worth noting that, in practice, $D_{\mathrm{c},i-1}$ is typically much smaller than h_i, and thus, the steady-state error is expected not to be large, particularly when the initial condition for speed is close to the leader's equilibrium speed or, at least, an estimate $\hat{D}_{\mathrm{c},i-1}$ of actual communication delay $D_{\mathrm{c},i-1}$ is available.

Remark 2: Note that the conditions in the statements of Theorem 1 do not depend on the delays values. To better understand the structure of our controller that allows simultaneous actuation and communication delays compensation we proceed as follows. We re-formulate the control laws (5) as[2]

$$u_i(t) = K_i p_i(t) + K_i \left(\mathrm{e}^{\Gamma_i D} \begin{bmatrix} \sigma_i(t) \\ 0 \\ \Delta v_{i-1}(t) \\ 0 \\ \Delta a_{i-1}(t) \end{bmatrix} + \int_{t-D}^{t} \mathrm{e}^{\Gamma_i (t-\theta)} B_{1i} \Delta u_{i-1}(\theta)\, d\theta \right), \tag{12}$$

[2]Using (18), relation (5) is written as $u_i(t) = K_i q_i(t) + \frac{\tau_i \alpha_i}{h_i} \sigma_i(t)$. Using (7), from definitions (8) and (14)–(17) we get $q_i(t) = \mathrm{e}^{\Gamma_i D} \tilde{x}_i(t) + \mathrm{e}^{\Gamma_i D} \begin{bmatrix} 0 & 0 & \Delta v_{i-1}(t) & 0 & \Delta a_{i-1}(t) \end{bmatrix}^T + \int_{t-D}^{t} \mathrm{e}^{\Gamma_i (t-\theta)} B_i u_i(\theta)\, d\theta + \int_{t-D}^{t} \mathrm{e}^{\Gamma_i (t-\theta)} B_{1i} u_{i-1}(\theta)\, d\theta + \int_{t-D}^{t} \mathrm{e}^{\Gamma_i (t-\theta)} B_{1i} \Delta u_{i-1}(\theta)\, d\theta$. With definitions (13), (A.5), and noting that $K_i \mathrm{e}^{\Gamma_i D} \begin{bmatrix} \sigma_i(t) & 0 & 0 & 0 & 0 \end{bmatrix}^T = \frac{\tau_i \alpha_i}{h_i} \sigma_i(t)$, we get (12).

where

$$p_i(t) = \mathrm{e}^{\Gamma_i D} \tilde{x}_i(t) + \int_{t-D}^{t} \mathrm{e}^{\Gamma_i (t-\theta)} B_i u_i(\theta)\, d\theta + \int_{t-D}^{t} \mathrm{e}^{\Gamma_i (t-\theta)} B_{1i} u_{i-1}(\theta)\, d\theta, \tag{13}$$

$$\tilde{x}_i = \begin{bmatrix} s_i & v_i & v_{i-1} & a_i & a_{i-1} \end{bmatrix}^T, \tag{14}$$

$$\Delta v_{i-1}(t) = v_{i-1,\mathrm{m}}(t) - v_{i-1}(t), \tag{15}$$

$$\Delta a_{i-1}(t) = a_{i-1,\mathrm{m}}(t) - a_{i-1}(t), \tag{16}$$

$$\Delta u_{i-1}(s) = u_{i-1,\mathrm{m}}(s) - u_{i-1}(s), \quad s \in [t-D, t], \tag{17}$$

$$K_i = \begin{bmatrix} \frac{\tau_i \alpha_i}{h_i} & -\tau_i(\alpha_i + b_i) & \tau_i b_i & \tau_i c_i & 0 \end{bmatrix}. \tag{18}$$

The terms (15)–(17) in parentheses of (12) are viewed as error due to the communication delays $D_{\mathrm{c},i-1}$, because in the nominal case $D_{\mathrm{c},i-1} = 0$, terms (15)–(17) are zero and p_i is the exact predictor of $\tilde{x}_i$, D-time units in advance. Structure of (12) reveals the reason for which Theorem 1 implies that communication delay does not affect individual vehicle stability. As it is evident from (12), communication delay affects only the feedforward and not the feedback terms in each ego vehicle's controller, i.e., it affects only measurements of the preceding vehicle's states; while the part $K_i p_i$ achieves input delay compensation and individual vehicle stability. This is consistent with the fact that the predictor-feedback controller is input-to-state stable (see, for example, [2], [15]) with respect to exogenous inputs and the states of the preceding vehicle, namely σ_i, Δv_{i-1}, Δa_{i-1}, and Δu_{i-1}, act as such, when viewed from the ego vehicle's dynamics perspective.

Remark 3: We next proceed to explaining how string stability is achieved. The states σ_i, Δv_{i-1}, and Δa_{i-1} involved in (12), for $t \geq \max_i \{D_{\mathrm{c},i-1}\}$ satisfy the following dynamics

$$\dot{\sigma}_i(t) = \Delta v_{i-1}(t), \tag{19}$$

$$\Delta \dot{v}_{i-1}(t) = \Delta a_{i-1}(t), \tag{20}$$

$$\Delta \dot{a}_{i-1}(t) = -\frac{1}{\tau_{i-1}} \Delta a_{i-1}(t) + \frac{1}{\tau_{i-1}} \Delta u_{i-1}(t-D). \tag{21}$$

Then, we define the new signal $\bar{p}_i$, as the predictor of states σ_i, Δv_{i-1}, and Δa_{i-1}, over a D-time units horizon, treating Δu_{i-1} as input, in the following manner

$$\bar{p}_i(t) = \mathrm{e}^{\bar{\Gamma}_i D} \hat{x}_i(t) + \int_{t-D}^{t} \mathrm{e}^{\bar{\Gamma}_i (t-\theta)} \bar{B}_{1i} \Delta u_{i-1}(\theta) d\theta, \tag{22}$$

where

$$\bar{p}_i = \begin{bmatrix} \bar{p}_{i,1} \\ \bar{p}_{i,2} \\ \bar{p}_{i,3} \end{bmatrix}, \quad \hat{x}_i = \begin{bmatrix} \sigma_i \\ \Delta v_{i-1} \\ \Delta a_{i-1} \end{bmatrix}, \tag{23}$$

$$\bar{B}_{1i} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau_{i-1}} \end{bmatrix}, \quad \bar{\Gamma}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_{i-1}} \end{bmatrix}. \tag{24}$$

Substituting (22) in the parentheses of (12) we get

$$u_i(t) = K_i p_i(t) + K_i \begin{bmatrix} \bar{p}_{i,1} \\ 0 \\ \bar{p}_{i,2} \\ 0 \\ \bar{p}_{i,3} \end{bmatrix}. \tag{25}$$

This article has been accepted for publication in IEEE Transactions on Intelligent Vehicles. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TIV.2024.3355334

Thus, (12) is written for $t \geq \max_i \{D_{c,i-1}\}$ as

$$\begin{aligned} u_i(t) = & \frac{\tau_i \alpha_i}{h_i} s_i(t+D) - \tau_i(\alpha_i + b_i) v_i(t+D) \\ & + \tau_i b_i v_{i-1}(t+D) + \tau_i c_i a_i(t+D) + \frac{\tau_i \alpha_i}{h_i} \sigma_i(t+D) \\ & + \tau_i b_i \left(v_{i-1,\mathrm{m}}(t+D) - v_{i-1}(t+D)\right). \end{aligned} \tag{26}$$

Hence, for $t \geq \max_i \{D_{c,i-1}\}$ we have

$$\begin{aligned} u_i(t) = & \tau_i \alpha_i \left(\frac{\bar{s}_i(t+D)}{h_i} - v_i(t+D) \right) \\ & + \tau_i b_i \left(v_{i-1,\mathrm{m}}(t+D) - v_i(t+D)\right) + \tau_i c_i a_i(t+D), \end{aligned} \tag{27}$$

where

$$\bar{s}_i(t) = s_i(t) + \sigma_i(t). \tag{28}$$

Noting that $\dot{\bar{s}}_i(t) = v_{i-1,\mathrm{m}}(t) - v_i(t)$, one could observe comparing (27) with the nominal controller (4) the following. First, all involved states are predicted by D-time units for input delay compensation. Second, communication delay is compensated through aiming at regulation of the ego's vehicle speed v_i to $v_{i-1,\mathrm{m}}$ rather than v_{i-1}. This is evident by the fact that $v_{i-1,\mathrm{m}}$ is involved in (27) (instead of v_{i-1}), together with the fact that $\bar{s}_i$ satisfies $\dot{\bar{s}}_i = v_{i-1,\mathrm{m}} - v_i$, which is viewed as the counterpart of s_i, under communication delay. One of the key aspects of the proposed controller, which makes it beneficial to string stability, is that the controller aims at regulating v_i to $v_{i-1,\mathrm{m}}$, while, simultaneously, aiming at regulating $\bar{s}_i$ (and not s_i) to $h_i v_i$. In other words, the controller regulates both speed and position of vehicle i taking into account the past (by $D_{c,i-1}$) speed and position (note that $\bar{s}_i$ could be viewed as $x_{i-1}(t - D_{c,i-1}) - x_i(t)$) of vehicle $i-1$, because this is the available information for the reference trajectory to vehicle i at time t (and it depends on the past preceding vehicle's speed/position). This, in a way, aligns the objectives of speed and spacing regulation, i.e., we regulate both, accounting for delayed information, rather than regulating speed but not spacing, using delayed information. This matching/synchronization is beneficial for string stability, because each vehicle's controller reacts uniformly, with respect to time, to speed and spacing deviations. Moreover, the way that the effect of the current speed of vehicle $i-1$ is being canceled from the spacing dynamics is by subtracting, via σ_i (having units of spacing), the term v_{i-1}. We note here that it is anticipated that, this characteristic of the controller, may not be beneficial for safety, due to the fact that the controller reacts to past and not to the current speed/spacing of the preceding vehicle.

Remark 4: The first two conditions of Theorem 1 come from the Routh-Hurwitz criterion and they are a prerequisite for string stability of the platoon. While the remaining two conditions are derived from the string stability criterion in speed error propagation. Feasibility of simultaneous satisfaction of the four conditions in Theorem 1 is explained noting, for example, that, since α_i and b_i are positive, the first three conditions can be satisfied with a proper choice of $\frac{1}{\tau_i} - c_i$ (via a proper choice of c_i); while the last condition can be satisfied, subsequently, with a proper choice of α_i and b_i.

IV. Numerical illustration of String Stability

In this section, we numerically analyze the string stability properties of the closed-loop system, according to Theorem 1. The transfer function $G_i = \frac{V_i}{V_{i-1}}$, which corresponds to the closed-loop systems described by equations (1)–(3), (5)–(10), along with choices (made for simplicity of illustration)

$$\alpha_i = -h_i p_i^3, \tag{29}$$

$$b_i = h_i p_i^3 + 3p_i^2, \tag{30}$$

$$c_i = \frac{1}{\tau_i} + 3p_i, \tag{31}$$

for some $p_i < 0$ and all i, is determined as

$$G_i(s) = \frac{V_i(s)}{V_{i-1}(s)} = \frac{-p_i^3 + p_i^2(p_i h_i + 3)s}{(s-p_i)^3} \mathrm{e}^{-sD_{c,i-1}}. \tag{32}$$

The numerical performance of the predictor-feedback CACC design (5) is showcased, focusing on $\mathcal{L}_2$ string stability definition in relation to (32). Fig. 2 depicts $\sup_\omega |G_i(j\omega)|$ as a function of p_i and h_i, where G_i is defined in (32). The conditions in Theorem 1, reduce to condition $h_i^2 p_i^2 + 6h_i p_i + 6 < 0$, which should hold to guarantee string stability. In Fig. 2, the region between the red curves indicates where condition $h_i^2 p_i^2 + 6h_i p_i + 6 < 0$ is satisfied.

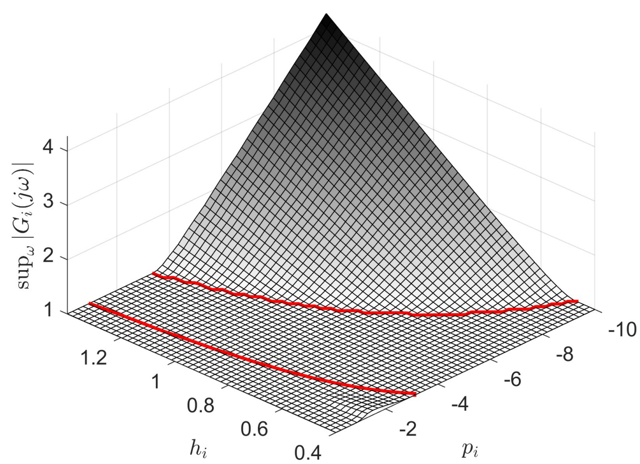

Fig. 2. The values of function $\sup_\omega |G_i(j\omega)|$ corresponding to transfer function (32) for heterogeneous vehicles, for different values of time-headway h_i and control parameter p_i.

In fact, string stability in $\mathcal{L}_p$, $p \in [1, +\infty]$, is also established. This follows based on the facts that $G_i(0) = 1$ and that (32) corresponds to a non-negative impulse response (see, e.g., [11]). As inferred from [29] (Theorem 5; case Type D-1), the validity of the latter is confirmed when the subsequent condition holds

$$-\frac{1}{p_i} \geq -\frac{h_i}{p_i}\left(p_i + \frac{3}{h_i}\right) \geq 0, \tag{33}$$

which can be also written as $-\frac{3}{h_i} \leq p_i \leq -\frac{2}{h_i}$ (that also guarantees $h_i^2 p_i^2 + 6h_i p_i + 6 < 0$). Note that all conditions for string stability do not depend on communication delay $D_{c,i-1}$ (or actuation delay D), demonstrating the delay-compensating property of our design.

V. Simulation Results

In this section first, the performance of the actuation/communication delays-compensating controller (5) is demonstrated, followed by a comparison with the predictor-feedback CACC approach in [26], which does not achieve communication delay compensation. Moreover, to illustrate the efficiency of predictor-feedback CACC with integral action in more practical scenarios, we utilize real traffic data from NGSIM.

A. Predictor-Feedback CACC with Simultaneous Compensation of Actuator/Communication Delays

At first we demonstrate the performance of the actuation/communication delays-compensating predictor-feedback CACC law. We consider a heterogeneous platoon of ten vehicles in order to make the numerical example more practical. For a heterogeneous platoon of ten vehicles with third-order dynamics given by (1)–(3), we consider a case in which $\tau_i = 0.1s$, $i = 1, 2, 6, 9$; $\tau_i = 0.2s$, $i = 0, 3, 5$; and $\tau_i = 0.25s$, $i = 4, 7, 8$. The desired time-headways are $h_{i,\mathrm{des}} = 0.75$, $i = 3, 4, 7, 9$; $h_{i,\mathrm{des}} = 0.9$, $i = 2, 5$; $h_{i,\mathrm{des}} = 1.2$, $i = 1, 6, 8$. The actuation delay is set to $D = 0.7$ and communication delays are $D_{\mathrm{c},i-1} = 0.1$, $i = 1, 4, 6$; $D_{\mathrm{c},i-1} = 0.15$, $i = 5, 8$; $D_{\mathrm{c},i-1} = 0.2$, $i = 3$; $D_{\mathrm{c},i-1} = 0.25$, $i = 2, 9$; and $D_{\mathrm{c},i-1} = 0.35$, $i = 7$. Following Remark 1, we assume that the communication delay is known. To address steady-state error, we employ in (5) time-headways $h_i = h_{\mathrm{des},i} - D_{\mathrm{c},i-1}$ (all h_i, $i = 1, 2, ..., 9$, satisfy the conditions in Theorem 1; see Fig. 2) and choose $\sigma_{i_0} = -\int_{-D_{\mathrm{c},i-1}}^{0} v_{i-1_0}(s)ds$ for all vehicles. Moreover, zero, steady-state spacing tracking errors are achieved as $\lim_{t\to+\infty} s_i(t) = h_{i,\mathrm{des}}v^*$, $i = 1, 2, ..., N$ (see Remark 1). We choose control gains according to (29)–(31) with $p_i = \frac{-2.5}{h_i}$, $i = 1, 2, ..., 9$ which satisfy the conditions in Theorem 1. Moreover, we consider a scenario in which $a_{i-1}(s) = 0, s \in [-D_{\mathrm{c},i-1}, 0]$ and $u_i(s) = 0$, $s \in [-D - D_{\mathrm{c},i-1}, 0)$ for each vehicle i. While we set $v_{i_0} = 15\left(\frac{m}{s}\right)$, $i = 1, 2, ..., 9$ and $v_{\mathrm{l}_0} = \frac{4v_{i_0}}{5} = 12\left(\frac{m}{s}\right)$; $v_{\mathrm{l}}(s) = 12, s \in [-D_{\mathrm{c},0}, 0]$ and $v_{i-1}(s) = 15, s \in [-D_{\mathrm{c},i-1}, 0]$, $i = 2, ..., 9$; $s_{i_0} = h_{\mathrm{des},i}v_{i_0} = h_{\mathrm{des},i} \times 15\ m$, $i = 2, 3, ..., 9$, $s_{1_0} = 16\ m$. Furthermore, the leading vehicle performs both deceleration and acceleration maneuvers.

Note that, here, we consider a scenario in which the initial condition for the speed of the leading vehicle ($12\left(\frac{m}{s}\right)$) is smaller than the initial speed of vehicle no. 1 ($15\left(\frac{m}{s}\right)$), i.e., the vehicle immediately behind the leader; while their respective initial spacing is $16\ m$. Thus, this scenario may appear in cases in which a vehicle (taking the role of the leader) cuts in, in front of the platoon (e.g., as a result of lane-changing from an adjacent lane) at a lower speed and short distance; or when a vehicle in the platoon changes lane to avoid a slowly moving vehicle in front (that takes the role of the leader after the lane-changing maneuver). Such a realistic correspondence, between the practical scenario of a vehicle cutting in the platoon and the simulation test using initial conditions not at equilibrium (with the leading vehicle having lower initial speed than vehicle no. 1), has been also considered in, for example, [16] (Scenario 3 in Section 2.2). In addition, the sharp deceleration maneuver of the leading vehicle in the same scenario (at $t = 20\ s$) could be also considered as related to a practical scenario of a vehicle changing lane in front of the leading vehicle with lower speed, thus causing the leading vehicle to decelerate abruptly. Such a correspondence, between the practical scenario of a vehicle cutting-in in front of the platoon and the simulation scenario of a sharp deceleration maneuver of the leading vehicle, has been also considered in, for example, [38] (Scenario 1 in Section 4.1).

As depicted in Fig. 3, the speed and acceleration responses to these maneuvers by the leading vehicle exhibit characteristics devoid of oscillations and overshoot. This desirable outcome is attributed to the obtained impulse response positivity and $\mathcal{L}_\infty$ string stability, respectively. These attributes are guaranteed for the corresponding transfer functions (32), subject to the condition (33). Furthermore, it is interesting to note that all states diverged with the nominal control law (4) in the presence of actuation/communication delays.

We note that if communication delays are not known exactly then we could still employ the choices $h_i = h_{i,\mathrm{des}} - \hat{D}_{\mathrm{c},i-1}$ and $\sigma_{i_0} = -\int_{-\hat{D}_{\mathrm{c},i-1}}^{0} v_{i-1_0}(s)ds$, with an estimate $\hat{D}_{\mathrm{c},i-1}$ of $D_{\mathrm{c},i-1}$. It is anticipated that steady-state, spacing tracking errors would remain small. The only case in which steady-state spacing errors would be large is when $D_{\mathrm{c},i-1}$ are both completely unknown and large which, in practice, may not be as realistic.

B. Predictor-Feedback Without Compensation of Communication Delay

In Fig. 4 we show the response of the heterogeneous platoon of ten vehicles under the predictor-feedback CACC law from [26] for system (1)–(3) that are given by

$$\bar{u}_i(t) = \frac{\tau_i\alpha_i}{h_i}\bar{q}_{i,1}(t) - \tau_i(\alpha_i + b_i)\bar{q}_{i,2}(t) + \tau_i b_i\bar{q}_{i,3}(t) + \tau_i c_i\bar{q}_{i,4}(t), \tag{34}$$

$$\bar{q}_i(t) = \mathrm{e}^{\Gamma_i D}\tilde{\bar{x}}_i(t) + \int_{t-D}^{t}\mathrm{e}^{\Gamma_i(t-\theta)}B_i\bar{u}_i(\theta)\,d\theta + \int_{t-D}^{t}\mathrm{e}^{\Gamma_i(t-\theta)}B_{1i}\bar{u}_{i-1,\mathrm{m}}(\theta)\,d\theta, \tag{35}$$

where

$$\bar{q}_i = \begin{bmatrix}\bar{q}_{i,1}\\ \bar{q}_{i,2}\\ \bar{q}_{i,3}\\ \bar{q}_{i,4}\\ \bar{q}_{i,5}\end{bmatrix}, \quad \tilde{\bar{x}}_i = \begin{bmatrix}s_i\\ v_i\\ v_{i-1}\\ a_i\\ a_{i-1,\mathrm{m}}\end{bmatrix}. \tag{36}$$

Control law (34) aims at only input delay compensation, but does not address communication delay. String stability under (34) is robust to the presence of small communication delay, as it is shown in [26]. We consider the scenario in which τ_i, D, and $D_{\mathrm{c},i-1}$ are the same with Section V-A and the desired time-headways are $h_i = 0.75$, $i = 3, 4, 7, 9$; $h_i = 0.9$, $i = 2, 5$; and $h_i = 1.2$, $i = 1, 6, 8$. We choose control gains according to (29)–(31) with $p_i = \frac{-2.5}{h_i}$, $i = 1, 2, ..., 9$, which satisfy the stability and string stability requirements when $D_{\mathrm{c},i-1} = 0$

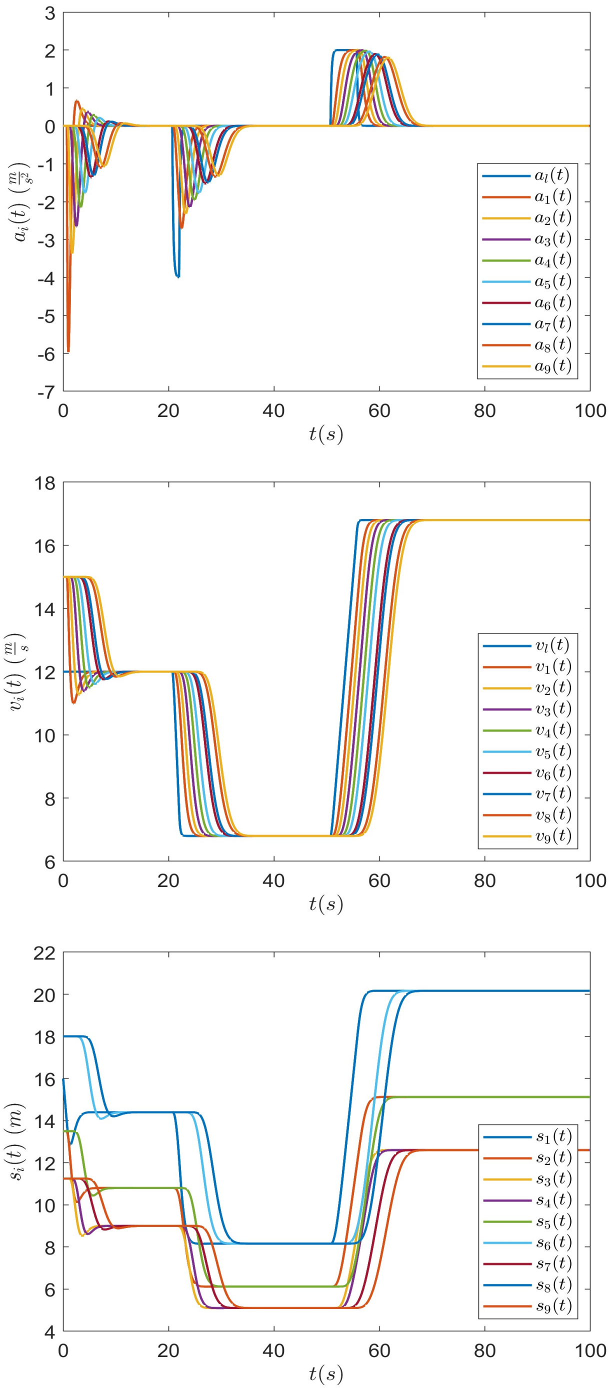

Fig. 3. Acceleration (top), speed (middle), and spacing (bottom) of ten vehicles, with dynamics described by (1)–(3), where $D = 0.7$, $\tau_i = 0.1s$, $i = 1, 2, 6, 9$; $\tau_i = 0.2s$, $i = 0, 3, 5$; and $\tau_i = 0.25s$, $i = 4, 7, 8$, following a leader that performs an acceleration/deceleration maneuver, under the CACC laws (5), where $D_{c,i-1} = 0.1$, $i = 1, 4, 6$; $D_{c,i-1} = 0.15$, $i = 5, 8$; $D_{c,i-1} = 0.2$, $i = 3$; $D_{c,i-1} = 0.25$, $i = 2, 9$; and $D_{c,i-1} = 0.35$, $i = 7$. The desired time-headways are $h_{i,\mathrm{des}} = 0.75$, $i = 3, 4, 7, 9$; $h_{i,\mathrm{des}} = 0.9$, $i = 2, 5$; $h_{i,\mathrm{des}} = 1.2$, $i = 1, 6, 8$; while control parameters are chosen according to (29)–(31) with $p_i = \frac{-2.5}{h_i}$ and $h_i = h_{i,\mathrm{des}} - D_{c,i-1}$. Initial conditions are $v_{i_0} = 15\left(\frac{m}{s}\right)$, $i = 1, 2, ..., 9$, $v_{l_0} = \frac{4v_{i_0}}{5} = 12\left(\frac{m}{s}\right)$; $s_{i_0} = h_{i,\mathrm{des}} v_{i_0} = h_{i,\mathrm{des}} \times 15\ m$, $i = 2, 3, ..., 9$, $s_{1_0} = 16\ m$; $\sigma_{i_0} = -\int_{-D_{c,i-1}}^{0} v_{i-1_0}(s)ds$ and $u_{i0} \equiv 0$, for $i = 1, 2, ..., 9$.

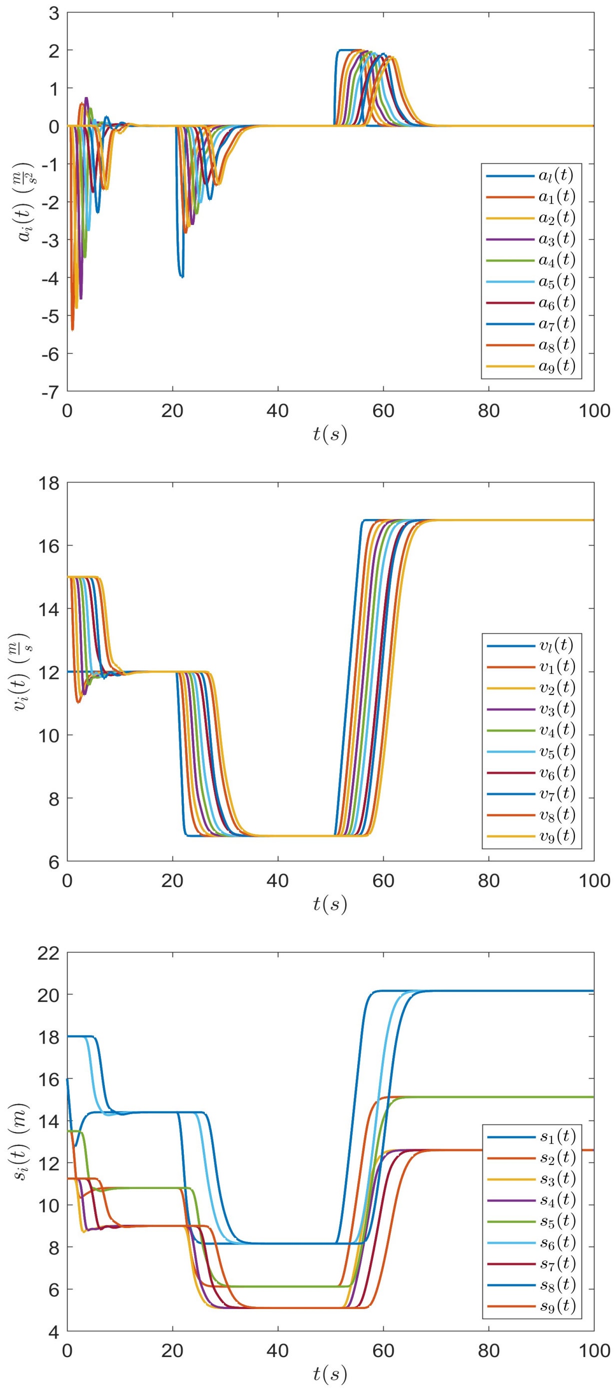

Fig. 4. Acceleration (top), speed (middle), and spacing (bottom) of ten vehicles, with dynamics described by (1)–(3), where $D = 0.7$, $\tau_i = 0.1s$, $i = 1, 2, 6, 9$; $\tau_i = 0.2s$, $i = 0, 3, 5$; and $\tau_i = 0.25s$, $i = 4, 7, 8$, following a leader that performs an acceleration/deceleration maneuver, under the CACC laws in [26] (see also (34)–(36)), where $D_{c,i-1} = 0.1$, $i = 1, 4, 6$; $D_{c,i-1} = 0.15$, $i = 5, 8$; $D_{c,i-1} = 0.2$, $i = 3$; $D_{c,i-1} = 0.25$, $i = 2, 9$; and $D_{c,i-1} = 0.35$, $i = 7$. The desired time-headways are $h_i = 0.75$, $i = 3, 4, 7, 9$; $h_i = 0.9$, $i = 2, 5$; and $h_i = 1.2$, $i = 1, 6, 8$; while control parameters are chosen according to (29)–(31) with $p_i = \frac{-2.5}{h_i}$. Initial conditions are $v_{i_0} = 15\left(\frac{m}{s}\right)$, $i = 1, 2, ..., 9$, $v_{l_0} = \frac{4v_{i_0}}{5} = 12\left(\frac{m}{s}\right)$; $s_{i_0} = h_i v_{i_0} = h_i \times 15\ m$, $i = 2, 3, ..., 9$, $s_{1_0} = 16\ m$; and $u_{i_0} \equiv 0$, for $i = 1, 2, ..., 9$.

(see Fig. 1-top in [26]). Since we consider the same scenario with Section V-A we choose identical initial conditions for a_i, v_i, u_i; while we set the initial spacing for $i = 2, 3, ..., 9$ to the corresponding equilibria for this this case, namely, $s_{i_0} = h_i v_{i_0} = h_i \times 15\ m$, and set $s_{1_0} = 16\ m$. The respective responses are, in general, more oscillatory. In addition, the response of vehicle 7 is string unstable, for the given values of h_7 and $D_{\mathrm{c},6}$ (see Fig. 3 in [26]), which results in overshoot in the respective response to leader's maneuvers.

C. Validation With NGSIM Data

In Fig. 5 we present the results of applying the predictor-feedback CACC law of Section V-A on NGSIM dataset. We extract reconstructed data from [21] to demonstrate the controller's performance in real maneuvering of the leading vehicle, considering that the leading vehicle's trajectory is taken from the real trajectory of vehicle no. 1601. This vehicle's trajectory is selected because it involves interesting dynamics with several acceleration/deceleration cycles. In fact, NGSIM data involve vehicles' trajectories that are taken from traffic in heavily congested conditions [21]. Indeed, from Fig. 5 (top and middle plots) it can be observed that the leading vehicle's (and thus, also the rest of the vehicles') trajectory features oscillations, as result of appearance of stop-and-go waves (evident in congested traffic flow); while its speed varies between around $15\left(\frac{m}{s}\right)$ and $4\left(\frac{m}{s}\right)$ (also a feature of congested traffic flow conditions).

We consider a heterogeneous platoon of five vehicles in order to make the numerical example more accessible and to more clearly illustrate the benefits of (5) in more practical scenarios. One difference from Section V-A is the predictor-feedback law for the first vehicle. Because we assume that the leading vehicle's dynamics satisfy $\dot{v}_{\mathrm{l}}(t) = u_{\mathrm{l}}(t - D)$ (and not the third-order system (1)–(3)) we have to modify slightly the predictor-feedback law for the first vehicle. In this case, for implementation of the predictor-feedback law, we set $u_{\mathrm{l}}(s)$, $s \geq 0$, using the practical command data, which are obtained from NGSIM dataset and we also set $u_{\mathrm{l}}(s) = 0$, for $s \in [-D - D_{\mathrm{c},0}, 0)$. Furthermore v_{l} is computed from the model $\dot{v}_{\mathrm{l}}(t) = u_{\mathrm{l}}(t - D)$, where $u_{\mathrm{l}}(t) = a_{\mathrm{l}}(t)$, $t \geq 0$, and $a_{\mathrm{l}}(t)$ are the NGSIM values of acceleration. This scenario could correspond to a case of a leading vehicle that is connected/automated or only connected, in which the control input commands $u_{\mathrm{l}}(t)$ (desired acceleration) affect the vehicle with a delay D; while these commands are transmitted to the following vehicle with communication delay $D_{\mathrm{c},0}$. We also note here that we do not validate the design in scenarios in which, for example, the leading vehicle's dynamics satisfy $\dot{v}_{\mathrm{l}}(t) = u_{\mathrm{l}}(t)$. The reason is that, because the leading vehicle's dynamics in such a case would not involve input delay, one would have to properly modify the predictor-feedback law for vehicle $i = 1$, for obtaining an implementable formula for the predictor state of v_{l}. This could be done, for example, as in [19], assuming constant speed for the leader. We do not investigate this scenario here, because this would imply that we also validate the design in such type of model mismatches, rather than validating it only accounting for real, leading vehicle's trajectories, which is our current scope here.

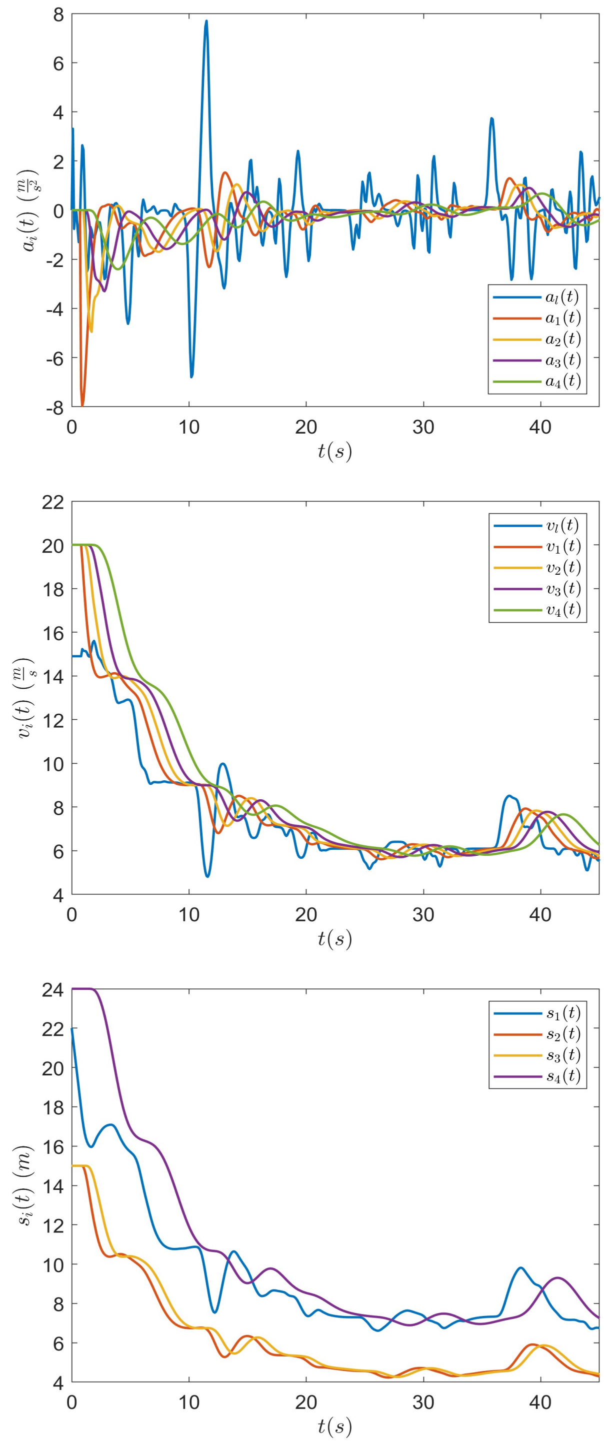

Fig. 5. Acceleration (top), speed (middle), and spacing (bottom) of five vehicles, where $D = 0.7$, $\tau_i = 0.1s$, $i = 1, 4$; $\tau_i = 0.2s$, $i = 0, 2$; and $\tau_i = 0.25s$, $i = 3$, following a leader whose trajectory is obtained from the trajectory of vehicle no. 1601 in the NGSIM data, under the CACC laws (5), where $D_{\mathrm{c},i-1} = 0.1$, $i = 1, 3, 4$; $D_{\mathrm{c},i-1} = 0.2$, $i = 2$. The desired time-headways are $h_{i,\mathrm{des}} = 1.2$, $i = 1, 4$; $h_{i,\mathrm{des}} = 0.75$, $i = 2, 3$; while control parameters are chosen according to (29)–(31) with $p_i = \frac{-2.5}{h_i}$ and $h_i = h_{i,\mathrm{des}} - D_{\mathrm{c},i-1}$, with $\sigma_{i_0} = -\int_{-D_{\mathrm{c},i-1}}^{0} v_{i-1_0}(s)ds$. Initial conditions are $v_{i_0} = 20\left(\frac{m}{s}\right)$, $i = 1, 2, 3, 4$, $v_{\mathrm{l}_0} = 14.9\left(\frac{m}{s}\right)$; $s_{i_0} = h_{i,\mathrm{des}} v_{i_0} = h_{i,\mathrm{des}} \times 20\ m$, $i = 2, 3, 4$, $s_{1_0} = 22\ m$; and $u_{i0} \equiv 0$, for $i = 1, 2, 3, 4$.

In this scenario we consider a case in which $\tau_i = 0.1s$, $i = 1, 4$; $\tau_i = 0.2s$, $i = 0, 2$; and $\tau_i = 0.25s$, $i = 3$. The desired time-headways are $h_{i,\mathrm{des}} = 1.2$, $i = 1, 4$; $h_{i,\mathrm{des}} = 0.75$, $i = 2, 3$. The actuation delay is set to $D = 0.7$ and communication delays are $D_{\mathrm{c},i-1} = 0.1$, $i = 1, 3, 4$; $D_{\mathrm{c},i-1} = 0.2$, $i = 2$. Following a similar approach to Section V-A, we assume that the communication delay is known, so we employ in (5) time-headways $h_i = h_{\mathrm{des},i} - D_{\mathrm{c},i-1}$, $i = 1, 2, 3, 4$ and choose $\sigma_{i_0} = -\int_{-D_{\mathrm{c},i-1}}^{0} v_{i-1_0}(s)ds$ for all vehicles to address steady-state spacing error. Moreover, we choose control gains according to (29)–(31) with $p_i = \frac{-2.5}{h_i}$, $i = 1, 2, 3, 4$, which satisfy the conditions in Theorem 1. We set $a_i(s) = 0, s \in [-D_{\mathrm{c},i-1}, 0]$ and $u_i(s) = 0$, $s \in [-D - D_{\mathrm{c},i-1}, 0)$ for vehicles $i = 1, 2, 3, 4$. While we also set $v_{i_0} = 20\left(\frac{m}{s}\right)$, $i = 1, 2, 3, 4$ and $v_{\mathrm{l}_0} = 14.9\left(\frac{m}{s}\right)$ (to match the initial speed of vehicle 1601 from NGSIM data); $v_{\mathrm{l}}(s) = 14.9, s \in [-D_{\mathrm{c},0}, 0]$ and $v_{i-1}(s) = 20, s \in [-D_{\mathrm{c},i-1}, 0]$, $i = 2, 3, 4$; $s_{i_0} = h_{\mathrm{des},i} v_{i_0} = h_{\mathrm{des},i} \times 20\ m$, $i = 2, 3, 4$, $s_{1_0} = 22\ m$. Fig. 5 illustrates that the performance of the predictor-feedback CACC law with integral action (5) is preserved even in more realistic traffic scenarios.

VI. Conclusions

In the present paper, we design a predictor-feedback CACC law with integral action, which achieves simultaneous actuation and communication delays compensation. We consider heterogeneous platoons with vehicles whose dynamics are described by a linear, third-order model with delayed actuation. The control design developed achieves string stability with respect to speed errors propagation, individual vehicle stability, and zero steady-state tracking errors. We provide constructive proof strategies that rely on a combination of an input-output approach and on deriving explicit solutions of the closed-loop systems. We demonstrate numerically the string stability conditions obtained and we provide simulation results for a platoon of ten vehicles, considering a realistic scenario of a vehicle cutting in the platoon and performing acceleration/deceleration maneuvers. We also validate the performance of the design developed in simulation, using real traffic data to describe the trajectory of the leading vehicle.

Appendix A

In order to studying stability and string stability of speed error propagation, we first compute the transfer functions

$$G_i(s) = \frac{V_i(s)}{V_{i-1}(s)}, \quad i = 1, ..., N, \tag{A.1}$$

viewing as input the preceding vehicle's speed and as output the current vehicle's speed. Taking Laplace transform of the predictor states (7) we get

$$\begin{aligned} Q_i(s) = \mathrm{e}^{\Gamma_i D} \bar{X}_i(s) + M_{1,i}(s) U_i(s) \\ + M_{2,i}(s) U_{i-1}(s) \mathrm{e}^{-s D_{\mathrm{c},i-1}}, \end{aligned} \tag{A.2}$$

where

$$M_{1,i}(s) = (sI_{5\times5} - \Gamma_i)^{-1}\left(I_{5\times5} - \mathrm{e}^{\Gamma_i D}\mathrm{e}^{-sD}\right)B_i, \tag{A.3}$$

$$\begin{aligned} M_{2,i}(s) = &(sI_{5\times5} - \Gamma_i)^{-1} \\ &\times \left(I_{5\times5} - \mathrm{e}^{\Gamma_i D}\mathrm{e}^{-sD}\right)B_{1i}, \end{aligned} \tag{A.4}$$

$$\mathrm{e}^{\Gamma_i D} = \begin{bmatrix} 1 & -D & D & E_{14} & E_{15} \\ 0 & 1 & 0 & E_{24} & 0 \\ 0 & 0 & 1 & 0 & \tau_{i-1}\left(1 - \mathrm{e}^{\frac{-D}{\tau_{i-1}}}\right) \\ 0 & 0 & 0 & \mathrm{e}^{\frac{-D}{\tau_i}} & 0 \\ 0 & 0 & 0 & 0 & \mathrm{e}^{\frac{-D}{\tau_{i-1}}} \end{bmatrix}, \tag{A.5}$$

$$E_{14} = \tau_i^2\left(1 - \frac{D}{\tau_i} - \mathrm{e}^{\frac{-D}{\tau_i}}\right), \tag{A.6}$$

$$E_{15} = \tau_{i-1}^2\left(\mathrm{e}^{\frac{-D}{\tau_{i-1}}} + \frac{D}{\tau_{i-1}} - 1\right), \tag{A.7}$$

$$E_{24} = \tau_i\left(1 - \mathrm{e}^{\frac{-D}{\tau_i}}\right). \tag{A.8}$$

Using (8), (10), (A.3), and (A.4) we get

$$\begin{aligned} (sI_{5\times5} - \Gamma_i)^{-1} &= \begin{bmatrix} s & 1 & -1 & 0 & 0 \\ 0 & s & 0 & -1 & 0 \\ 0 & 0 & s & 0 & -1 \\ 0 & 0 & 0 & s + \frac{1}{\tau_i} & 0 \\ 0 & 0 & 0 & 0 & s + \frac{1}{\tau_{i-1}} \end{bmatrix}^{-1} \\ &= \frac{1}{s^2}\begin{bmatrix} s & -1 & 1 & -\frac{\tau_i}{s\tau_i + 1} & -\frac{\tau_{i-1}}{s\tau_{i-1} + 1} \\ 0 & s & 0 & \frac{s\tau_i}{s\tau_i + 1} & 0 \\ 0 & 0 & s & 0 & \frac{s\tau_{i-1}}{s\tau_{i-1} + 1} \\ 0 & 0 & 0 & \frac{s^2\tau_i}{s\tau_i + 1} & 0 \\ 0 & 0 & 0 & 0 & \frac{s^2\tau_{i-1}}{s\tau_{i-1} + 1} \end{bmatrix}, \end{aligned} \tag{A.9}$$

$$M_{1,i}(s) = \begin{bmatrix} M_{11,i}(s) & M_{21,i}(s) & 0 & M_{41,i}(s) & 0 \end{bmatrix}^{\mathrm{T}}, \tag{A.10}$$

$$M_{2,i}(s) = \begin{bmatrix} m_{11,i}(s) & 0 & m_{31,i}(s) & 0 & m_{51,i}(s) \end{bmatrix}^{\mathrm{T}}, \tag{A.11}$$

where

$$\begin{aligned} M_{11,i}(s) = &\frac{\mathrm{e}^{-sD}\left(\tau_i - \tau_i e^{\frac{-D}{\tau_i}}\right)}{s^2\tau_i} \\ &+ \frac{\mathrm{e}^{-sD}\left(\tau_i^2\mathrm{e}^{\frac{-D}{\tau_i}} + D\tau_i - \tau_i^2\right)}{s\tau_i} \\ &+ \frac{\tau_i\left(\mathrm{e}^{-\frac{D}{\tau_i}}\mathrm{e}^{-sD} - 1\right)}{s^2\tau_i(s\tau_i + 1)}, \end{aligned} \tag{A.12}$$

$$M_{21,i}(s) = -\frac{\mathrm{e}^{-sD}(\tau_i - \tau_i\mathrm{e}^{\frac{-D}{\tau_i}})}{s\tau_i} - \frac{\tau_i\left(\mathrm{e}^{\frac{-D}{\tau_i}}\mathrm{e}^{-sD} - 1\right)}{s\tau_i(s\tau_i + 1)}, \tag{A.13}$$

$$M_{41,i}(s) = -\frac{\mathrm{e}^{\frac{-D}{\tau_i}}\mathrm{e}^{-sD} - 1}{s\tau_i + 1}, \tag{A.14}$$

$$\begin{aligned} m_{11,i}(s) =& -\frac{\mathrm{e}^{-sD}\left(\tau_{i-1}-\tau_{i-1}\mathrm{e}^{\frac{-D}{\tau_{i-1}}}\right)}{s^2\tau_{i-1}} \\ &-\frac{\mathrm{e}^{-sD}\left(\tau_{i-1}^2\mathrm{e}^{\frac{-D}{\tau_{i-1}}}+D\tau_{i-1}-\tau_{i-1}^2\right)}{s\tau_{i-1}} \\ &-\frac{\tau_{i-1}\left(\mathrm{e}^{-\frac{D}{\tau_{i-1}}}\mathrm{e}^{-sD}-1\right)}{s^2\tau_{i-1}(s\tau_{i-1}+1)}, \end{aligned} \tag{A.15}$$

$$\begin{aligned} m_{31,i}(s) =& -\frac{\mathrm{e}^{-sD}\left(\tau_{i-1}-\tau_{i-1}\mathrm{e}^{\frac{-D}{\tau_{i-1}}}\right)}{s\tau_{i-1}} \\ &-\frac{\tau_{i-1}\left(\mathrm{e}^{\frac{-D}{\tau_{i-1}}}\mathrm{e}^{-sD}-1\right)}{s\tau_{i-1}(s\tau_{i-1}+1)}, \end{aligned} \tag{A.16}$$

$$m_{51,i}(s) = -\frac{\mathrm{e}^{\frac{-D}{\tau_{i-1}}}\mathrm{e}^{-sD}-1}{s\tau_{i-1}+1}. \tag{A.17}$$

Using the i-th vehicle's model (1)–(3) we obtain

$$\begin{aligned} \begin{bmatrix} S_i(s) \\ V_i(s) \\ A_i(s) \end{bmatrix} =& \left(sI_{3\times 3}-\hat{\Gamma}_i\right)^{-1}\left(\begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau_i} \end{bmatrix}\mathrm{e}^{-sD}U_i(s)\right. \\ &\left. + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} V_{i-1}(s)\right), \end{aligned} \tag{A.18}$$

where

$$\hat{\Gamma}_i = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_i} \end{bmatrix}, \tag{A.19}$$

and hence

$$\begin{bmatrix} S_i(s) \\ V_i(s) \\ A_i(s) \end{bmatrix} = \begin{bmatrix} -\frac{1}{s^2(s\tau_i+1)} \\ \frac{1}{s(s\tau_i+1)} \\ \frac{1}{s\tau_i+1} \end{bmatrix}\mathrm{e}^{-sD}U_i(s) + \begin{bmatrix} \frac{1}{s} \\ 0 \\ 0 \end{bmatrix} V_{i-1}(s). \tag{A.20}$$

Using control laws (5), together with (A.2)–(A.17), we arrive at

$$\begin{aligned} U_i(s) =& \frac{\tau_i\alpha_i}{h_i}S_i(s) - \left(\tau_i(\alpha_i+b_i)+\frac{D\tau_i\alpha_i}{h_i}\right)V_i(s) \\ &-\frac{\tau_i\alpha_i}{sh_i}V_{i-1}(s) \\ &+\left(\tau_ib_i+\frac{D\tau_i\alpha_i}{h_i}+\frac{\tau_i\alpha_i}{sh_i}\right)\mathrm{e}^{-D_{c,i-1}s}V_{i-1}(s) \\ &+\left(\tau_ic_i\mathrm{e}^{\frac{-D}{\tau_i}}-\tau_i(\alpha_i+b_i)\left(\tau_i-\tau_i\mathrm{e}^{\frac{-D}{\tau_i}}\right)\right. \\ &\left.-\frac{\tau_i\alpha_i}{h_i}\left(\tau_i^2\mathrm{e}^{\frac{-D}{\tau_i}}+D\tau_i-\tau_i^2\right)\right)A_i(s) \\ &+\left(\tau_ib_i\left(\tau_{i-1}-\tau_{i-1}\mathrm{e}^{\frac{-D}{\tau_{i-1}}}\right)+\frac{\tau_i\alpha_i}{h_i}\left(\tau_{i-1}^2\mathrm{e}^{\frac{-D}{\tau_{i-1}}}\right.\right. \\ &\left.\left.+D\tau_{i-1}-\tau_{i-1}^2\right)\right)\mathrm{e}^{-D_{c,i-1}s}A_{i-1}(s) \\ &+g_{1,i}(s)U_i(s)+g_{2,i}(s)U_{i-1}(s)\mathrm{e}^{-D_{c,i-1}s}, \end{aligned} \tag{A.21}$$

where

$$\begin{aligned} g_{1,i}(s) =& \left(\frac{\tau_i\alpha_i}{h_i}M_{11,i}(s)-\tau_i(\alpha_i+b_i)M_{21,i}(s)\right. \\ &\left.+\tau_ic_iM_{41,i}(s)\right), \end{aligned} \tag{A.22}$$

$$g_{2,i}(s) = \left(\frac{\tau_i\alpha_i}{h_i}m_{11,i}(s)+\tau_ib_im_{31,i}(s)\right). \tag{A.23}$$

Hence, substituting (A.20) in (A.21) we derive $\frac{U_i}{U_{i-1}}$, which, multiplying it by $\frac{s\tau_{i-1}+1}{s\tau_i+1}$, gives

$$G_i(s) = \frac{\left(b_is+\frac{\alpha_i}{h_i}\right)\mathrm{e}^{-D_{c,i-1}s}}{s^3+\left(\frac{1}{\tau_i}-c_i\right)s^2+(\alpha_i+b_i)s+\frac{\alpha_i}{h_i}}. \tag{A.24}$$

String stability in $\mathcal{L}_2$ is guaranteed when $|G_i(j\omega)|\le 1$, for all $\omega\ge 0$. The condition is satisfied for $\omega=0$ since $|G_i(0)|=1$. Moreover, from (A.24) we have

$$G_i(j\omega) = \frac{f_{1,i}(\omega)+jf_{2,i}(\omega)}{f_{3,i}(\omega)+jf_{4,i}(\omega)}\mathrm{e}^{-D_{c,i-1}j\omega}, \tag{A.25}$$

$$f_{1,i}(\omega) = \frac{\alpha_i}{h_i}, \tag{A.26}$$

$$f_{2,i}(\omega) = b_i\omega, \tag{A.27}$$

$$f_{3,i}(\omega) = \omega^2\left(\mathrm{c}_i-\frac{1}{\tau_i}\right)+\frac{\alpha_i}{h_i}, \tag{A.28}$$

$$f_{4,i}(\omega) = \omega(\alpha_i+b_i)-\omega^3. \tag{A.29}$$

By using the fact that $\sup_\omega|\mathrm{e}^{-D_{c,i-1}j\omega}|=1$, the condition for string stability becomes $f_{1,i}(\omega)^2+f_{2,i}(\omega)^2<f_{3,i}(\omega)^2+f_{4,i}(\omega)^2$, $\omega>0$, $i=1,...,N$, and hence, after straightforward computations, we get the following condition that has to hold for all $\omega>0$ and $i=1,...,N$

$$\omega^6+\omega^4f_{5,i}(\omega)+\omega^2f_{6,i}(\omega)>0, \tag{A.30}$$

where

$$f_{5,i}(\omega) = \left(c_i-\frac{1}{\tau_i}\right)^2-2(\alpha_i+b_i), \tag{A.31}$$

$$f_{6,i}(\omega) = \alpha_i\left(\alpha_i+2b_i+\frac{2}{h_i}\left(c_i-\frac{1}{\tau_i}\right)\right). \tag{A.32}$$

Relation (A.30) holds for all $\omega>0$, under the conditions on the parameters a_i, b_i, c_i, τ_i, h_i of Theorem 1.

We next show that boundedness of all states is achieved. Using the delay-compensating property of predictor feedback (see e.g., [2]), we have, using (26), that for $t\ge\max\left\{D,\max_i\{D_{c,i-1}\}\right\}=\bar{D}$ (note that, due to linearity of systems (1)–(3) and controllers (5), no finite escape time phenomenon can appear for $t<\bar{D}$) it holds

$$\begin{aligned} \begin{bmatrix} \dot{s}_i(t) \\ \dot{v}_i(t) \\ \dot{a}_i(t) \end{bmatrix} =& \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ \frac{a_i}{h_i} & -(a_i+b_i) & c_i-\frac{1}{\tau_i} \end{bmatrix}\begin{bmatrix} s_i(t) \\ v_i(t) \\ a_i(t) \end{bmatrix} \\ &+\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}v_{i-1}(t)+\begin{bmatrix} 0 \\ 0 \\ b_i \end{bmatrix}v_{i-1,\mathrm{m}}(t)+\begin{bmatrix} 0 \\ 0 \\ \frac{a_i}{h_i} \end{bmatrix}\sigma_i(t), \end{aligned} \tag{A.33}$$

$$\dot{\sigma}_i(t) = v_{i-1,\mathrm{m}}(t)-v_{i-1}(t). \tag{A.34}$$

The solution to (A.33), (A.34) is given as

$$\begin{aligned}\begin{bmatrix}s_i(t)\\ v_i(t)\\ a_i(t)\end{bmatrix} =&\mathrm{e}^{\bar{A}_i\left(t-\bar{D}\right)}\begin{bmatrix}s_i\left(\bar{D}\right)\\ v_i\left(\bar{D}\right)\\ a_i\left(\bar{D}\right)\end{bmatrix}\\ &+\int_{\bar{D}}^{t}\mathrm{e}^{\bar{A}_i(t-s)}\left(\begin{bmatrix}1\\0\\0\end{bmatrix}v_{i-1}(s)+\begin{bmatrix}0\\0\\b_i\end{bmatrix}v_{i-1,\mathrm{m}}(s)\right.\\ &\left.+\begin{bmatrix}0\\0\\\frac{a_i}{h_i}\end{bmatrix}\sigma_i(s)\right)ds, \end{aligned}\tag{A.35}$$

$$\sigma_i(t)=\sigma_i\left(\bar{D}\right)+\int_{\bar{D}}^{t}\left(v_{i-1,\mathrm{m}}(s)-v_{i-1}(s)\right)ds, \tag{A.36}$$

where $\bar{A}_i=\begin{bmatrix}0&-1&0\\0&0&1\\\frac{a_i}{h_i}&-(a_i+b_i)&c_i-\frac{1}{\tau_i}\end{bmatrix}$. Under the conditions in Theorem 1, $\bar{A}_i$ always has eigenvalues with strictly negative real part, which means that the states s_i, v_i, a_i remain bounded, provided that σ_i and v_{i-1} are bounded. We establish next the boundedness of σ_1 under the assumption that the leader's speed, denoted as v_0, is bounded by, say, M_{v_0}, i.e., $|v_0(s)|\leq \mathrm{M}_{v0}$, for all $s\geq -D_{\mathrm{c},0}$. We derive

$$\begin{aligned}\sigma_1(t)=&\sigma_1(0)+\int_0^t\left(v_0(s-D_{\mathrm{c},0})-v_0(s)\right)ds\\ =&\sigma_1(0)+\int_{-D_{\mathrm{c},0}}^{t-D_{\mathrm{c},0}}v_0(s)ds-\int_0^t v_0(s)ds,\end{aligned}\tag{A.37}$$

and thus,

$$\sigma_1(t)=\sigma_1(0)+\int_{-D_{\mathrm{c},0}}^{0}v_0(s)ds-\int_{t-D_{\mathrm{c},0}}^{t}v_0(s)ds. \tag{A.38}$$

Considering the assumption on the leader's speed being bounded we can derive that

$$\int_{t-D_{\mathrm{c},0}}^{t}|v_0(s)|\,ds\leq\int_{t-D_{\mathrm{c},0}}^{t}\mathrm{M}_{v_0}ds=\mathrm{M}_{v_0}D_{\mathrm{c},0}. \tag{A.39}$$

Thus, considering (A.38), (A.39), it follows that σ_1 is uniformly bounded, with $|\sigma_1(t)|\leq\mathrm{M}_{\sigma_1}$, where $\mathrm{M}_{\sigma_1}=\sigma_1(0)+2\mathrm{M}_{v_0}D_{\mathrm{c},0}$, $t\geq 0$. For showing boundedness of v_1 we proceed as follows. By using (A.35) for $i=1$, with the fact that $\left|\mathrm{e}^{\bar{A}_1\left(t-\bar{D}\right)}\right|\leq k_1\mathrm{e}^{-\lambda_1\left(t-\bar{D}\right)}$, for some positive constants k_1, λ_1 (because A_1 is Hurwitz, see, e.g., [17]) we get for $t\geq\bar{D}$

$$|v_1(t)|\leq r_{1,1}(t)+r_{2,1}(t)\leq\bar{v}_1(t), \tag{A.40}$$

where

$$r_{1,1}(t)=k_1\mathrm{e}^{-\lambda_1\left(t-\bar{D}\right)}\left(\left|s_1\left(\bar{D}\right)\right|+\left|v_1\left(\bar{D}\right)\right|+\left|a_1\left(\bar{D}\right)\right|\right), \tag{A.41}$$

$$\begin{aligned}r_{2,1}(t)=&\int_{\bar{D}}^{t}k_1\mathrm{e}^{-\lambda_1(t-s)}\left(|v_0(s)|+|b_1v_{0,\mathrm{m}}(s)|\right.\\ &\left.+\left|\frac{\alpha_1}{h_1}\sigma_1(s)\right|\right)ds,\end{aligned}\tag{A.42}$$

$$\bar{v}_1(t)=r_{1,1}(t)+\frac{(1+b_1)\,k_1}{\lambda_1}\mathrm{M}_{v_0}+\frac{k_1\alpha_1}{\lambda_1h_1}\mathrm{M}_{\sigma_1}. \tag{A.43}$$

Relations (A.40)–(A.43) imply that v_1 is uniformly bounded with $|v_1(s)|\leq\mathrm{M}_{v_1}$, $s\geq -D_{\mathrm{c},1}$. This assertion is based on the observation that v_1 is bounded by constant terms $\frac{(1+b_1)k_1}{\lambda_1}\mathrm{M}_{v_0}$, $\frac{k_1\alpha_1}{\lambda_1h_1}\mathrm{M}_{\sigma_1}$, and an exponentially decaying term, as well as using the fact that $v_1(s)$, $s\in[-D_{\mathrm{c},1},0]$ is also bounded (by assumption). Then we need to show that σ_2 is uniformly bounded. We derive

$$\begin{aligned}\sigma_2(t)=&\sigma_2(0)+\int_0^t\left(v_1(s-D_{\mathrm{c},1})-v_1(s)\right)ds\\ =&\sigma_2(0)+\int_{-D_{\mathrm{c},1}}^{0}v_1(s)ds-\int_{t-D_{\mathrm{c},1}}^{t}v_1(s)ds.\end{aligned}\tag{A.44}$$

Following a similar argument as for the boundedness of σ_1 and since

$$\int_{t-D_{\mathrm{c},1}}^{t}|v_1(s)|\,ds\leq\int_{t-D_{\mathrm{c},1}}^{t}\mathrm{M}_{v_1}ds=\mathrm{M}_{v_1}D_{\mathrm{c},1}, \tag{A.45}$$

we conclude that σ_2 is uniformly bounded. We next show that v_2 is bounded. Similarly to derivation of (A.40)–(A.43), there exist some positive constants k_2 and λ_2, such that for $t\geq\bar{D}$

$$|v_2(t)|\leq r_{1,2}(t)+r_{2,2}(t)\leq\bar{v}_2(t), \tag{A.46}$$

where

$$r_{1,2}(t)=k_2\mathrm{e}^{-\lambda_2\left(t-\bar{D}\right)}\left(\left|s_2\left(\bar{D}\right)\right|+\left|v_2\left(\bar{D}\right)\right|+\left|a_2\left(\bar{D}\right)\right|\right), \tag{A.47}$$

$$\begin{aligned}r_{2,2}(t)=&\int_{\bar{D}}^{t}k_2\mathrm{e}^{-\lambda_2(t-s)}\left(|v_1(s)|+|b_2v_{1,\mathrm{m}}(s)|\right.\\ &\left.+\left|\frac{\alpha_2}{h_2}\sigma_2(s)\right|\right)ds,\end{aligned}\tag{A.48}$$

$$\bar{v}_2(t)=r_{1,2}(t)+\frac{(1+b_2)\,k_2}{\lambda_2}\mathrm{M}_{v_1}+\frac{k_2\alpha_1}{\lambda_2h_2}\mathrm{M}_{\sigma_2}. \tag{A.49}$$

This pattern continues iteratively up to $i=N$. Consequently, we can deduce by induction that v_i and σ_i, $i=1,...,N$, are bounded. From (A.35) and the fact that the $\bar{A}_i$ matrices are Hurwitz we conclude that the system's states s_i and a_i are also bounded.

Next, we demonstrate how control law (5) regulates s_i, v_i, a_i, and σ_i, and we also compute their steady-state values. Regulation follows starting with $i=1$ and considering $v_0\equiv v^*$ as the leader's speed having constant value. This assumption leads to bounded limits for each σ_i and, subsequently, results in finite limits for v_i, s_i, and a_i. To clarify this, we begin with σ_1, which is constant (given a constant v_0). This enables us to recursively compute finite limits for v_1, s_1, and a_1, based on (A.35) and the fact that $\bar{A}_1$ is Hurwitz. Then, using (A.44), together with the continuity of v_1 (for $t\geq\bar{D}$) and the fact that v_1 has a finite limit, we obtain that σ_2 has a finite limit. By considering (A.35) and the finite limit of states v_1 and σ_2, as well as the fact that $\bar{A}_2$ is Hurwitz, we deduce that v_2, s_2, and a_2 have finite limits. This process continues recursively, leading to the conclusion that s_i, v_i, a_i, and σ_i, $i=1,2,...,N$, have finite limits. Moreover, we can derive $\lim_{t\to+\infty}\dot{s}_i(t)=\lim_{t\to+\infty}\dot{v}_i(t)=\lim_{t\to+\infty}\dot{a}_i(t)=0$, $i=1,2,...,N$. This is established by applying, for example, Barbalat's lemma, (see, e.g., [27]), given that s_i, v_i, and a_i have finite limits and by using (A.33), (A.34), which allow us to conclude boundedness of $\ddot{s}_i$, $\ddot{v}_i$, $\ddot{a}_i$. Subsequently, by using (A.33) we deduce that $\lim_{t\to+\infty} v_i(t) = \lim_{t\to+\infty} v_{i-1}(t) = v^*$, and $\lim_{t\to+\infty} v_{i-1,\mathrm{m}}(t) = \lim_{t\to+\infty} v_{i-1}(t) = v^*$. Moreover, we conclude that $\lim_{t\to+\infty} s_i(t) = \lim_{t\to+\infty} (h_i v_i(t) - \sigma_i(t))$, where

$$\begin{aligned}\lim_{t\to+\infty} \sigma_i(t) =& \sigma_i(0) + \int_{-D_{\mathrm{c},i-1}}^{0} v_{i-1_0}(s)ds \\ &- \lim_{t\to+\infty} \int_{-D_{\mathrm{c},i-1}}^{0} v_{i-1}(s+t)ds. \qquad \text{(A.50)}\end{aligned}$$

Due to the fact that v_{i-1} is a continuous function on the time interval $\left(\bar{D},\ +\infty\right)$ and uniformly bounded, we can derive

$$\lim_{t\to+\infty} \sigma_i(t) = \sigma_i(0) + \int_{-D_{\mathrm{c},i-1}}^{0} v_{i-1_0}(s)ds - D_{\mathrm{c},i-1} v^*, \qquad \text{(A.51)}$$

which completes the proof.

ACKNOWLEDGMENT

The authors thank Dr. Panagiotis Karafotis for fruitful discussions on the choice of the leading vehicle's trajectories from NGSIM data.

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