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Compensation of actuator dynamics governed by quasilinear hyperbolic PDEs[☆]

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A B S T R A C T

We present a methodology for stabilization of general nonlinear systems with actuator dynamics governed by a certain class of, quasilinear, first-order hyperbolic PDEs. Since for such PDE–ODE cascades the speed of propagation depends on the PDE state itself (which implies that the prediction horizon cannot be a priori known analytically), the key design challenge is the determination of the predictor state. We resolve this challenge and introduce a PDE predictor-feedback control law that compensates the transport actuator dynamics. Due to the potential formation of shock waves in the solutions of quasilinear, first-order hyperbolic PDEs (which is related to the fundamental restriction for systems with time-varying delays that the delay rate is bounded by unity), we limit ourselves to a certain feasibility region around the origin and we show that the PDE predictor-feedback law achieves asymptotic stability of the closed-loop system, providing an estimate of its region of attraction. Our analysis combines Lyapunov-like arguments and ISS estimates. Since it may be intriguing as to what is the exact relation of the cascade to a system with input delay, we highlight the fact that the considered PDE–ODE cascade gives rise to a system with input delay, with a delay that depends on past input values (defined implicitly via a nonlinear equation). The developed control design methodology is applied to the control of vehicular traffic flow at distant bottlenecks.

1. Introduction

1.1. Motivation

Numerous processes may be described by quasilinear, first-order hyperbolic Partial Differential Equations (PDEs) cascaded with nonlinear Ordinary Differential Equations (ODEs), such as, for example, communication networks (Espitia, Girard, Marchand, & Prieur, 2017), blood flow (Borsche, Colombo, & Garavello, 2010), sewer networks (Cunge, Holly, & Verwey, 1980), production systems (Gottlich, Herty, & Klar, 2005), vehicular traffic flow (Herty, Lebacque, & Moutari, 2009), piston dynamics (Lattanzio, Mauriz, & Piccoli, 2011), automotive engines (Depcik & Assanis, 2005; Jankovic & Magner, 2011; Kahveci & Jankovic, 2010), and oil drilling (Aamo, 2013; Hasan, Aamo, & Krstic, 2006) to name only a few (Richard, 2003). Despite their popularity, despite the fact that predictor-based control laws now exist for nonlinear systems with input delays that may depend on the ODE state (Bekiaris-Liberis & Krstic, 2013a,b,c,d; Cai & Krstic, 2015, 2016) as well as the uncontrolled- or controlled-boundary value of the PDE state (Bresch-Pietri, Chauvin, & Petit, 2012a,b, 2014; Diagne, Bekiaris-Liberis, & Krstic, 2017), and despite the existence of several results on boundary stabilization of quasilinear, first-order hyperbolic PDEs, such as, for example, Blandin, Litrico, Delle Monache, Piccoli, & Bayen (2017), Coron & Bastin (2016), Hu, Vazquez, Meglio, & Krstic, (2017), Krstic (1999), Prieur, Winkin, & Bastin (2008), Vazquez, Coron, Krstic, & Bastin (2011) and (2012), no result exists on the compensation of actuator dynamics governed by quasilinear, first-order hyperbolic PDEs for nonlinear systems.

1.2. Contributions

In this paper, we consider the problem of stabilization of nonlinear ODE systems through transport actuator dynamics governed by quasilinear, first-order hyperbolic PDEs. We develop a novel PDE predictor-feedback law, which compensates the PDE actuator dynamics. Since the speed of propagation depends on the PDE

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state itself, the key idea in our design is the construction of the PDE predictor state. This construction is by far non-trivial and cannot follow in a straightforward way employing the results from Diagne et al. (2017), which is perhaps the only available work dealing with the problem of complete compensation of an input-dependent input delay (note that the designs in Bresch-Pietri et al. (2012b), Bresch-Pietri et al. (2012a), 2014), do not aim at achieving complete delay compensation). The reason is that the transport speed in the class of systems considered in Diagne et al. (2017) depends only on the *uncontrolled-boundary* value of the PDE state rather than on the PDE state itself, as it is the case here.

Furthermore, we show that the PDE predictor-feedback design achieves local asymptotic stability in the C^1 norm of the actuator state. The reason for obtaining only a regional result, restricting the C^1 norm of the PDE state, is the possibility of appearance of multivalued solutions, or, in other words, the appearance of shock waves, in the solutions of quasilinear, first-order hyperbolic PDEs. We show, within our stability analysis, that this issue is avoided, limiting the C^1 norm of the solutions and the initial conditions. This limitation may alternatively be expressed as the fundamental limitation in stabilization of systems with time-varying input delays that the delay rate is bounded by unity—for the class of systems considered here, giving rise to an input delay that depends on the actuator state and its derivative, the satisfaction of this restriction is guaranteed by confining the size of the actuator state and its derivative. The proof of asymptotic stability in the C^1 norm of the actuator state is established employing Lyapunov-like arguments as well as Input-to-State Stability (ISS) estimates.

In order to make the presentation of our control design methodology accessible to both readers who are experts on PDEs and readers who are experts on delay systems we highlight the relation of the PDE–ODE cascade to a system with input delay that is defined implicitly through a nonlinear equation, which involves the input value at a time that depends on the delay itself, and, moreover, we present the predictor-feedback design in this representation as well.

We also present a numerical example of control of a vehicular traffic flow model, which accounts for the traffic flow dynamics at bottleneck areas (areas with lower flow capacity) located far downstream from the actuation point (for instance, where an on-ramp is located), see, for example, Herty et al. (2009) and Wang, Kosmatopoulos, Papageorgiou, & Papamichail (2014), to illustrate the proposed control design framework.

1.3. Organization

We start in Section 2 where we present the class of systems under consideration as well as the PDE predictor-feedback control design. We provide an alternative, delay system representation of the considered PDE–ODE cascade in Section 3. In Section 4 we prove the local asymptotic stability of the closed-loop system under the proposed controller. Simulation results of an example of vehicular traffic flow control at distant bottlenecks are presented in Section 5. Concluding remarks are provided in Section 6.

Notation: We use the common definition of class $\mathcal{K}$, $\mathcal{K}_\infty$ and $\mathcal{KL}$ functions from Khalil (2002). For an n-vector, the norm $|\cdot|$ denotes the usual Euclidean norm. For a scalar function $u \in C[0,1]$ we denote by $\|u(t)\|_C$ its respective maximum norm, i.e., $\|u(t)\|_C = \max_{x\in[0,1]}|u(x,t)|$. For a scalar function $u_x \in C[0,1]$ we denote by $\|u_x(t)\|_C$ its respective maximum norm, i.e., $\|u_x(t)\|_C = \max_{x\in[0,1]}|u_x(x,t)|$. For a vector valued function $p \in C[0,1]$ we denote by $\|p(t)\|_C$ its respective maximum norm, i.e., $\|p(t)\|_C = \max_{x\in[0,1]}\sqrt{p_1(x,t)^2+\cdots+p_n(x,t)^2}$. For a vector valued function $p_x \in C[0,1]$ we denote by $\|p_x(t)\|_C$ its respective maximum norm, i.e., $\|p_x(t)\|_C = \max_{x\in[0,1]}\sqrt{p_{1x}(x,t)^2+\cdots+p_{nx}(x,t)^2}$. We denote by $C^j(A;E)$ the space of functions that take values in E and have continuous derivatives of order j on A.

2. Problem formulation and predictor-feedback control design

We consider the following system

$$\dot{X}(t) = f\left(X(t), u(0,t)\right) \tag{1}$$

$$u_t(x,t) = v\left(u(x,t)\right) u_x(x,t) \tag{2}$$

$$u\left(1,t\right) = U(t), \tag{3}$$

where $X \in \mathbb{R}^n$ and $u \in \mathbb{R}$ are ODE and PDE states, respectively, $t \geq 0$ is time, $x \in [0,1]$ is spatial variable, U is control input, and $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is a continuously differentiable vector field that satisfies $f(0,0) = 0$.

The following assumptions are imposed on system (1)–(3).

Assumption 1. Function $v : \mathbb{R} \to \mathbb{R}_+$ is twice continuously differentiable and there exists a positive constant $\underline{v}$ such that the following holds

$$v\left(u\right) \geq \underline{v}, \quad \text{for all } u \in \mathbb{R}. \tag{4}$$

Assumption 2. System $\dot{X} = f\left(X, \omega\right)$ is strongly forward complete with respect to ω.

Assumption 3. There exists a twice continuously differentiable feedback law $\kappa : \mathbb{R}^n \to \mathbb{R}$, with $\kappa(0) = 0$, which renders system $\dot{X} = f\left(X, \kappa(X) + \omega\right)$ input-to-state stable with respect to ω.

Assumption 1 is a prerequisite for the well-posedness of the predictor state, which is defined in the next paragraph. It guarantees that transport is happening only in the direction away from the input, or, in other words (see also the discussion in the next section), it ensures that the input delay is positive as well as uniformly bounded. Assumption 2 (see, e.g., Angeli & Sontag, 1999) and Assumption 3 (see, e.g., Sontag & Wang, 1995) are standard ingredients of the predictor-feedback control design methodology (see, e.g., Bekiaris-Liberis & Krstic, 2013a; Krstic, 2009, 2010). The former implies that the state X of system (1) does not escape to infinity before the control signal U reaches it, no matter the size of the delay (see, e.g., Bekiaris-Liberis & Krstic, 2013a; Krstic, 2009, 2010), while the latter guarantees the existence of a nominal feedback law that renders system (1) input-to-state stable in the absence of the transport actuator dynamics (i.e., in the absence of the input delay).

The predictor-feedback control law for system (1)–(3) is given by

$$U(t) = \kappa\left(p\left(1,t\right)\right), \tag{5}$$

where for all $t \geq 0$

$$p\left(x,t\right) = X(t) + \int_0^x f\left(p(y,t), u(y,t)\right) \times \Gamma\left(u(y,t), u_y(y,t), y\right) dy, \quad x \in [0,1] \tag{6}$$

with[1]

$$\Gamma\left(u(x,t), u_x(x,t), x\right) = \frac{1}{v\left(u(x,t)\right)} - \frac{x v'\left(u(x,t)\right) u_x(x,t)}{v(u(x,t))^2}, \quad x \in [0,1]. \tag{7}$$

For implementing the predictor-feedback law (5)–(7), besides measurements of the ODE state $X(t)$ and the PDE state $u(x,t)$, $x \in [0,1]$, for all $t \geq 0$, the availability of the spatial derivative

[1] Note that Γ can be written as $\Gamma\left(u(x,t), u_x(x,t), x\right) = \frac{\partial \frac{x}{v(u(x,t))}}{\partial x}$.

of u, namely, $u_x(x,t)$, $x \in [0,1]$, for all $t \geq 0$, is required. The latter may be obtained either via direct measurements of u_x or by a numerical computation of u_x, employing the measurements of u. The implementation and approximation problems of predictor-feedback control laws are tackled, for example, in Karafyllis and Krstic (2017), Mondie and Michiels (2003) and Zhong (2004).

Feasibility Condition In order to guarantee the well-posedness of the predictor state (6) and the system the following feasibility condition on the closed-loop solutions and the initial conditions needs to be satisfied

$$-M < \frac{v'(u(x,t))\, u_x(x,t)}{v(u(x,t))} < 1, \quad \text{for all } x \in [0,1] \text{ and } t \geq 0, \tag{8}$$

for some $M > 0$.

As it is shown later on (in Section 4), this condition is fulfilled by appropriately restricting the size of the initial conditions only (see relation (27)).

In the next section we provide some explanatory remarks on the feasibility condition (8) and Assumption 1, capitalizing on the relation of the PDE–ODE cascade (1)–(3) to a system with a delayed-input-dependent input delay.

Example 1. To illustrate the control design and its implementation we present here a rather pedagogical example, which results in a predictor-feedback law defined explicitly in terms of X, u, and u_x. Consider an unstable, scalar linear system with actuator dynamics governed by a quasilinear, first-order hyperbolic PDE given by

$$\dot{X}(t) = X(t) + u(0,t) \tag{9}$$

$$u_t(x,t) = \left(u(x,t)^2 + 1\right) u_x(x,t) \tag{10}$$

$$u(1,t) = U(t). \tag{11}$$

System (9)–(11) satisfies all of Assumption 1–3 and a nominal control law may be chosen as $U(t) = -2X(t)$. Thus, the predictor-feedback control law is given by

$$U(t) = -2p(1,t), \tag{12}$$

where, exploiting the fact that $\Gamma = \frac{\partial \frac{x}{u(x,t)^2+1}}{\partial x}$ as well as the linearity of the system, the predictor state p, defined in (6), may be written in the present case as[2]

$$p(x,t) = e^{\frac{x}{u(x,t)^2+1}} \left(X(t) + u(0,t) + \int_0^x e^{-\frac{y}{u(y,t)^2+1}} u_y(y,t)dy \right) - u(x,t), \quad x \in [0,1]. \tag{13}$$

For the numerical computation of the integral in (13) we employ a simple composite left-endpoint rectangular rule, where the spatial derivates of u are numerically computed utilizing a forward finite difference scheme. We choose the initial conditions as

$$u(x,0) = 1, \quad \text{for all } x \in [0,1] \tag{14}$$

$$X(0) = -0.7. \tag{15}$$

In Fig. 1 we show the response of the system, whereas in Fig. 2 we show the control effort.

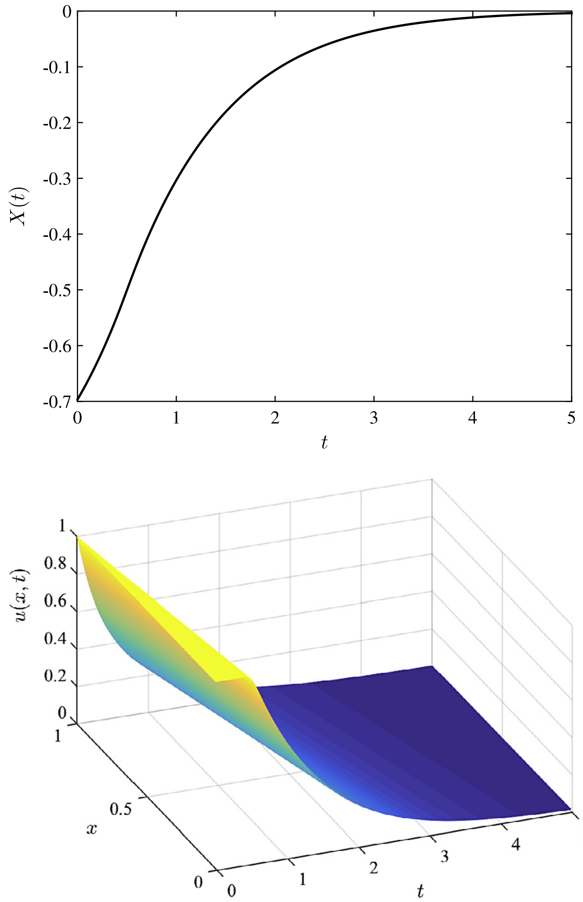

Fig. 1. Response of system (9)–(11) with initial conditions (14), (15) under the predictor-feedback law (12), (13).

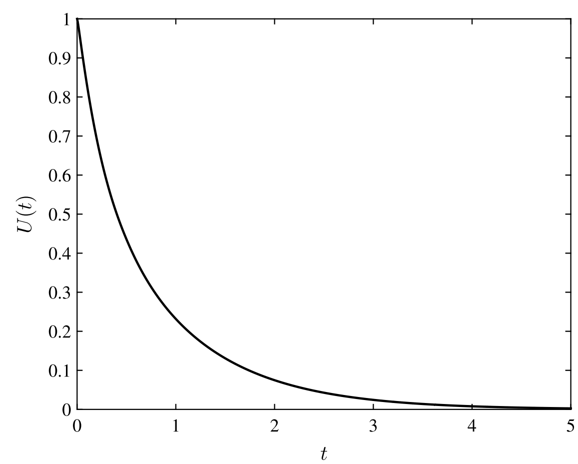

Fig. 2. Control effort (12), (13).

3. Relation to a system with delayed-input-dependent input delay

In this section, we highlight the fact that the PDE–ODE cascade (1)–(3) may be viewed as a nonlinear system with an input delay. The fact that the transport speed depends on the PDE state itself, gives rise to a delay that is defined implicitly through a nonlinear equation, which incorporates the value of the input at a time that depends on the delay itself.

[2] To see this note that, for the case of system (9)–(11), the predictor state p in (6) satisfies, for each t, the ODE in x given as $p_x(x,t) = (p(x,t)+u(x,t)) \frac{\partial \frac{x}{u(x,t)^2+1}}{\partial x}$, with initial condition $p(0,t) = X(t)$. Thus, solving this initial-value problem with respect to x we obtain $p(x,t) = e^{\int_0^x \frac{\partial \frac{y}{u(y,t)^2+1}}{\partial y} dy} X(t) + \int_0^x e^{\int_y^x \frac{\partial \frac{r}{u(r,t)^2+1}}{\partial r} dr} u(y,t) \frac{\partial \frac{y}{u(y,t)^2+1}}{\partial y} dy$. Expression (13) then follows evaluating the integral in the first term of this relation and employing one step of integration by parts in the integral in the second.

The reasons for emphasizing this alternative representation of system (1)–(3) are not merely pedagogical. Capitalizing on this relation, enables both, readers who are experts on PDEs and readers who are experts on delay systems, to digest the key conceptual ideas as well as the technical intricacies of our design and analysis methodologies, such as, for example, to better understand some of the inherent limitations of the stabilization problem for such systems (see Section 3.2). Moreover, this alternative point of view, offers to the designer two alternative control law representations (see Section 3.3), which may be very useful since, depending on the specific application, one representation may be more descriptive of the actual physical process as well as more suitable for implementation than the other (consider, for example, the case of control of traffic flow versus the case of control over a network).

3.1. Derivation of the delayed and prediction times

Employing the method of characteristics (for details, see, e.g., Courant & Hilbert, 1962), it can be shown, see, e.g., Petit, Creff, & Rouchon (1998),[3] that the following holds

$$u(0,t)=U\left(t-\frac{1}{v\left(u(0,t)\right)}\right). \tag{16}$$

Thus, defining the delayed time ϕ, i.e., the time at which the value of the control signal U that currently affects the system, namely, $u(0,t)$, was actually applied, as

$$\phi(t)=t-\frac{1}{v\left(u(0,t)\right)}, \tag{17}$$

we re-write system (1)–(3) as

$$\dot{X}(t)=f\left(X(t),U\left(\phi(t)\right)\right), \tag{18}$$

where ϕ is defined implicitly, for all $t\geq 0$, through relation

$$\phi(t)=t-\frac{1}{v\left(U\left(\phi(t)\right)\right)}. \tag{19}$$

The prediction time σ, i.e., the time at which the value of the control signal U currently applied, namely, $U(t)=u(1,t)$, will actually reach the system, is defined as the inverse function of ϕ, namely,

$$\sigma(t)=t+\frac{1}{v\left(U(t)\right)}. \tag{20}$$

The invertibility of ϕ is guaranteed when the derivative of (19), given by

$$\dot{\phi}(t)=\frac{1}{1-\frac{v'(U(\phi(t)))U'(\phi(t))}{v(U(\phi(t)))^2}}, \tag{21}$$

is positive for all $t\geq 0$, or, equivalently, when the derivative of (20), given by

$$\dot{\sigma}(t)=1-\frac{v'\left(U(t)\right)}{v(U(t))^2}U'(t), \tag{22}$$

is positive for all $t\geq 0$.

3.2. Interpretation of Assumption 1 and Condition (8)

From (19) it is evident that the positivity assumption of v guarantees that the delay is always positive, i.e., it guarantees the causality of system (18), and thus, also of system (1)–(3). Moreover, relation (4) guarantees the boundedness of the delay, i.e., it guarantees that the control signal eventually reaches the plant (18), and thus, also (1).

The interpretation of condition (8) is less obvious. When the derivative of the prediction (or the delayed) time is bounded and strictly positive both the prediction and delayed times are well-defined. Via (3), it is evident from (22) that this requirement is satisfied when condition (8) holds. In fact, condition (8) guarantees that the quasilinear first-order hyperbolic PDE (2), (3) exhibits smooth solutions and that the appearance of shock waves is avoided.

To see this, note that when the right-hand side of (8) is violated the derivative of the delayed time becomes infinite (or, equivalently, the derivative of the prediction time becomes zero), that is, the delay disappears instantaneously (with slope approaching negative infinity). This implies that the delayed time becomes a multi-valued function, which in turn is related to loss of regularity of the solutions to (2), (3) and the formation of a shock wave. Moreover, when u is bounded, the regularity assumption on v implies that the left-hand side of (8) may be violated when u_x reaches negative infinity. Thus, in terms of the delay system representation, the left-hand side of condition (8) guarantees that the time derivative of the prediction time cannot become infinite, and thus, the predictor state (6) remains well-posed (see also relation (25)).

3.3. Predictor-Feedback control design for the equivalent delay system

Defining

$$F(U)=\frac{1}{v(U)}, \tag{23}$$

the predictor-feedback control law for system (18) with an input delay defined via (19) is given by

$$U(t)=\kappa\left(P(t)\right), \tag{24}$$

where the predictor P is given for all $t\geq 0$ by

$$P(\theta)=X(t)+\int_{\phi(t)}^{\theta}\left(1+F'\left(U(s)\right)\dot{U}(s)\right)\times f\left(P(s),U(s)\right)ds,\quad \text{for all } \phi(t)\leq\theta\leq t. \tag{25}$$

The predictor-feedback control law (25) is implementable since, for all $t\geq 0$, it depends on the history of $U(s)$, over the window $\phi(t)\leq s\leq t$, the ODE state $X(t)$, which are assumed to be measured for all $t\geq 0$, as well as on $\dot{U}(s)$, over the window $\phi(t)\leq s\leq t$, which is assumed to either be measured directly or computed from the values of $U(s)$, $\phi(t)\leq s\leq t$. Moreover, the implementation of the predictor-feedback design requires the computation at each time step of the delayed time ϕ. This can either be performed by numerically solving relation (19), using the history of the actuator state, or by employing the following integral equation

$$\phi(\theta)=t-\int_{\theta}^{\sigma(t)}\frac{ds}{1+F'\left(U\left(\phi(s)\right)\right)U'\left(\phi(s)\right)},\quad \text{for all } t\leq\theta\leq\sigma(t), \tag{26}$$

where σ is defined in (20). The issue of implementation and approximation of nonlinear predictor feedbacks is addressed in detail in Karafyllis (2011), Karafyllis and Krstic (2014, 2017).

4. Stability analysis

Theorem 1. *Consider the closed-loop system consisting of the plant (1)–(3) and the control law (5)–(7). Under Assumptions 1–3, there*

[3] In detail, along the characteristic curves defined by $\frac{dt(x)}{dx}=-\frac{1}{v(u(x,t(x)))}$ the solution to (2) remains constant, and thus, integrating from $x=1$ and using (3) we obtain $u(x,t)=U(t_1)$, where $t_1-t=-\frac{1-x}{v(U(t_1))}$. Thus, setting $x=0$ we get $u(0,t)=U(t_1)$, where t_1 is defined as $t_1=t-\frac{1}{v(U(t_1))}$. For instance, relation (16) in Example 1 becomes $u(0,t)=U\left(t-\frac{1}{u(0,t)^2+1}\right)$.

exist a positive constant δ and a class $\mathcal{KL}$ function β such that for all initial conditions $X(0) \in \mathbb{R}^n$ and $u(\cdot, 0) \in C^1[0, 1]$ which satisfy

$$|X(0)| + \|u(0)\|_C + \|u_x(0)\|_C < \delta, \tag{27}$$

as well as the compatibility conditions

$$u(1,0) = \kappa(p(1,0)) \tag{28}$$

$$u_x(1,0) = \frac{\partial\kappa(p(1,0))}{\partial p} f(p(1,0), u(1,0)) \times \Gamma(u(1,0), u_x(1,0), 1), \tag{29}$$

there exists a unique solution to the closed-loop system with $X(t) \in C^1[0,\infty)$, $u(x,t) \in C^1([0,1]\times[0,\infty))$, and the following holds

$$\Omega(t) \le \beta(\Omega(0), t), \quad \text{for all } t \ge 0 \tag{30}$$

$$\Omega(t) = |X(t)| + \|u(t)\|_C + \|u_x(t)\|_C. \tag{31}$$

The proof of Theorem 1 is based on the following lemmas whose proofs can be found in Appendix.

Lemma 1. *The variable*

$$u(x,t) - \kappa(p(x,t)) = w(x,t), \tag{32}$$

where p is defined in (6), satisfies

$$w_t(x,t) = v(u(x,t))\, w_x(x,t) \tag{33}$$

$$w(1,t) = 0. \tag{34}$$

Moreover, system (1) can be written as

$$\dot{X}(t) = f(X(t), \kappa(X(t)) + w(0,t)). \tag{35}$$

Note that, in contrast to our previous work on predictor-feedback design, but similarly to Kharitonov's work (Kharitonov, 2014), the variable w is just viewed as a new variable, which is expressed in terms of the state (X, u) via (32), (6), (7), rather than as a transformation of the original state u. Thus, an inverse transformation is not required, which does not affect the analysis (see Lemma 4 and its proof in Appendix). The reason for this alternative point of view is that the expression for the potential inverse transformation would require the definition of an alternative, rather complex representation of the predictor state p that would depend on the new variable w, which would add unnecessary complexity in the analysis.

The next lemma establishes an asymptotic stability estimate for state variables $(X, w(x))$, $x \in [0, 1]$, exploiting the cascade structure of system (33)–(35).

Lemma 2. *There exists a class $\mathcal{KL}$ function β_w such that for all solutions of the system satisfying (8) the following holds*

$$\Omega_w(t) \le \beta_w(\Omega_w(0), t), \quad \text{for all } t \ge 0 \tag{36}$$

$$\Omega_w(t) = |X(t)| + \|w(t)\|_C + \|w_x(t)\|_C. \tag{37}$$

In Lemma 3–5, the equivalency of the C^1 norm between the original state variables $(X, u(x))$, $x \in [0, 1]$, and the state variables $(X, w(x))$, $x \in [0, 1]$, is established. The proofs of each of Lemmas 3 and 4, utilize different arguments and employ different assumptions. For this reason, the proof of the norm equivalency, between the original and the new state variables, is decomposed into three different lemmas.

Lemma 3. *There exists a class $\mathcal{K}_\infty$ function ρ_1 such that for all solutions of the system satisfying (8) the following holds*

$$\|p(t)\|_C + \|p_x(t)\|_C \le \rho_1(|X(t)| + \|u(t)\|_C), \quad \text{for all } t \ge 0. \tag{38}$$

Lemma 4. *There exists a class $\mathcal{K}_\infty$ function ρ_2 such that for all solutions of the system satisfying (8) the following holds*

$$\|p(t)\|_C + \|p_x(t)\|_C \le \rho_2(|X(t)| + \|w(t)\|_C), \quad \text{for all } t \ge 0. \tag{39}$$

Lemma 5. *There exist class $\mathcal{K}_\infty$ functions ρ_3 and ρ_4 such that for all solutions of the system satisfying (8) the following hold*

$$\Omega_w(t) \le \rho_3(\Omega(t)), \quad \text{for all } t \ge 0 \tag{40}$$

$$\Omega(t) \le \rho_4(\Omega_w(t)), \quad \text{for all } t \ge 0, \tag{41}$$

where Ω_w is defined in (37) and Ω is defined in (31).

An estimate of the region of attraction of the predictor-feedback control law (5)–(7) within the feasibility region, defined by condition (8), is derived in the next two lemmas.

Lemma 6. *There exists a positive constant δ_1 such that all of the solutions that satisfy*

$$|X(t)| + \|u(t)\|_C + \|u_x(t)\|_C < \delta_1, \quad \text{for all } t \ge 0, \tag{42}$$

also satisfy (8).

Lemma 7. *There exists a positive constant δ such that for all initial conditions of the closed-loop system (1)–(3), (5)–(7) that satisfy (27), the solutions of the system satisfy (42), and hence, satisfy (8).*

Proof of Theorem 1. Estimate (30) in Theorem 1 is proved combining Lemmas 2 and 5 with

$$\beta_u(s,t) = \rho_4(\beta_w(\rho_3(s), t)). \tag{43}$$

We show next the well-posedness of the system. We start by proving the well-posedness of the predictor

$$P(t) = p(1,t), \tag{44}$$

where p is defined in (6). Differentiating definition (6) with respect to t and using integration by parts in the integral, taking into account that p satisfies $p_t(x,t) = v(u(x,t))\, p_x(x,t)$ (see relation (A.1) in Appendix) and employing relations (3), (5) we obtain that

$$\dot{P}(t) = v(\kappa(P(t)))\, f(P(t), \kappa(P(t))) \times \Gamma(u(1,t), u_x(1,t), 1). \tag{45}$$

From the definition of Γ in (6) and (7), using (2) we obtain from (3), (5) that

$$\Gamma(u(1,t), u_x(1,t), 1) = \frac{1}{v(\kappa(P(t)))} - \frac{v'(\kappa(P(t)))\, u_t(1,t)}{v(\kappa(P(t)))^3}, \tag{46}$$

and thus, from (45) we arrive at

$$\frac{1}{v(\kappa(P(t)))} = \left(1 + \frac{v'(\kappa(P(t)))}{v(\kappa(P(t)))^2} \times \frac{\partial\kappa(P(t))}{\partial P} f(P(t), \kappa(P(t)))\right) \times \Gamma(u(1,t), u_x(1,t), 1). \tag{47}$$

Since condition (8) guarantees the positivity and boundedness of Γ (see also relations (A.43), (A.45) in Appendix), it follows from (47) and Assumption 1 that the term $1 + \frac{v'(\kappa(P(t)))\frac{\partial\kappa(P(t))}{\partial P} f(P(t),\kappa(P(t)))}{v(\kappa(P(t)))^2}$

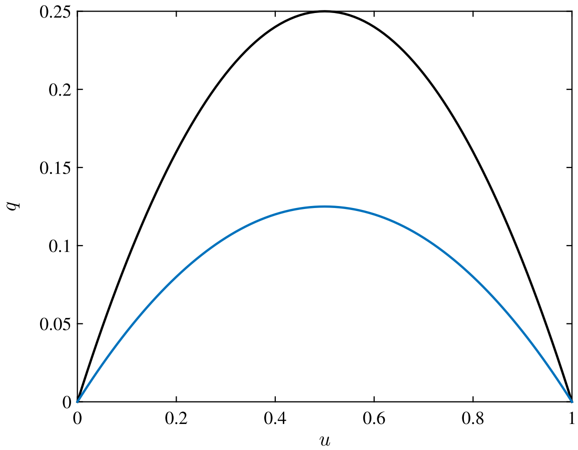

Fig. 3. Fundamental diagrams of a mainstream highway stretch (black line) and of a bottleneck area (blue line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

is positive. Hence, solving (47) with respect to Γ and substituting the resulting expression into (45) we arrive at

$$\dot{P}(t) = \frac{1}{1+\frac{v'(\kappa(P(t)))\frac{\partial\kappa(P(t))}{\partial P}f(P(t),\kappa(P(t)))}{v(\kappa(P(t)))^2}} \times f\left(P(t), \kappa\left(P(t)\right)\right). \tag{48}$$

Under the regularity assumptions on v and κ, which follow from Assumptions 1 and 3, respectively, one can conclude that the right-hand side of (48) is Lipschitz with respect to P. Therefore, the compatibility conditions (28), (29) guarantee that there exists a unique solution $P(t) \in C^1[0,\infty)$. Moreover, employing estimates (30), (38), one can conclude that δ in the statement of Theorem 1 can be chosen sufficiently small such that all the conditions of Theorem 1.1 in Li (1984, Chapter 5) are satisfied, and hence, the existence and uniqueness of $u(x,t) \in C^1([0,1]\times[0,\infty))$, which satisfies (2), (3), (5), follows (see also the discussion in, e.g., Coron & Bastin, 2016; Prieur et al., 2008). The fact that $u(x,t) \in C^1([0,1]\times[0,\infty))$ and the regularity properties of f imply from (1) the existence and uniqueness of $X(t) \in C^1[0,\infty)$.

5. Application to traffic flow control at distant bottlenecks

Ramp metering is successful in regulating traffic flow at bottleneck areas (i.e., areas that feature a significantly smaller capacity due to the presence of, for example, curvature, tunnel, narrowing, etc.) that are located close to the ramp-flow merging area. However, the performance of ramp metering-based traffic control algorithms may deteriorate considerably when the bottleneck area is located far downstream from the location of the actuated ramp, due to the large distance of the actuated ramp from the bottleneck area that is controlled (imposing a large, input-dependent delay) (Wang et al., 2014).

Consider a highway stretch with inlet at $x = 1$ and outlet at $x = 0$. Let u denote the density of vehicles within the stretch whose dynamics satisfy a conservation law equation of the form (2). For Greenshield's fundamental diagram (see, e.g., Claudel, 2010) with free-flow vehicles' velocity and maximum vehicles' density equal to unity the transport speed v is given by

$$v(u) = 1 - 2u, \tag{49}$$

which corresponds to the following relation (black line in Fig. 3) between the flow q and the density of vehicles within the stretch

$$q(u) = u(1-u). \tag{50}$$

At the end of the stretch there is a bottleneck area, e.g., a lane-drop, which imposes a 50% reduction in flow capacity (consider, e.g., the case of a two-lane highway stretch). The fundamental diagram relation at the bottleneck area is shown in Fig. 3 (blue line). Modeling the traffic flow dynamics at the bottleneck area with a cell that has half of the capacity of the mainstream, the dynamics of the density X at the bottleneck may be described by (1) with

$$f(X(t), u(0,t)) = \frac{1}{\Delta}\left(u(0,t)(1-u(0,t)) - \frac{1}{2}X(t)(1-X(t))\right), \tag{51}$$

where Δ is the length of the cell. The boundary condition at the inlet of the stretch is dictated by a mainstream inflow d as well as the inflow from a ramp r giving rise to the following equation

$$q(u(1,t)) = d(t) + r(t). \tag{52}$$

The ramp flow r is the manipulated variable to the overall system. In the considered scenarios we assume a constant mainstream inflow d.

The goal of the predictor-feedback control design is to regulate the density at the bottleneck area to an uncongested, constant equilibrium X^*, which is given by

$$X^* = \frac{1-\sqrt{1-8u^*(1-u^*)}}{2}, \tag{53}$$

where u^* denotes a constant equilibrium profile for the density at the stretch. The control input is chosen as

$$r(t) = \left(U(t)+u^*\right)(1-u(1,t)) - d, \tag{54}$$

which imposes a boundary condition of the form (3), where U is yet to be designed according to the predictor-feedback control design methodology.

The predictor-feedback law is chosen as

$$U(t) = -k\left(p(1,t) - X^*\right), \tag{55}$$

where the predictor $p(1,t)$ is given by (6), (7) where v and f are defined in (49) and (51), respectively. The nominal, delay-free closed-loop system satisfies

$$\dot{\tilde{X}}(t) = \frac{1}{\Delta}\tilde{X}(t)\left(X^* - \frac{1}{2} - k\left(1-2u^*\right)\right) + \frac{1}{\Delta}\left(\frac{1}{2}-k^2\right)\tilde{X}(t)^2, \tag{56}$$

where $\tilde{X} = X - X^*$, which can be made asymptotically stable choosing, e.g., $k = \frac{\sqrt{2}}{2}$. To see this note that, using (53), relation (56) can be then written as

$$\dot{\tilde{X}}(t) = -\frac{a}{\Delta}\tilde{X}(t), \tag{57}$$

where $a = \frac{\sqrt{1-8u^*(1-u^*)}+\sqrt{2}(1-2u^*)}{2}$.

We employ Rusanov's scheme to solve numerically the considered conservation law equation as well as a forward Euler scheme to solve numerically the considered ODE, see, e.g., Trangenstein (2007). Furthermore, we employ a left endpoint rule for the numerical approximation of the integral in the predictor-feedback control law as well as a finite-difference approximation for the numerical computation of the spatial derivative of the PDE state.

We choose $d = 0.1$, $u^* = 0.145$, and $\Delta = 0.1$ (that may correspond to a cell with length 1 km for a 10-km highway stretch). The initial conditions are chosen as $u(x,0) = 0.1$, for all $x \in [0,1]$ and $X(0) = 0.24$ (that corresponds to the equilibrium value of X). Note that from (53) we obtain $X^* = 0.46$, which is quite close to

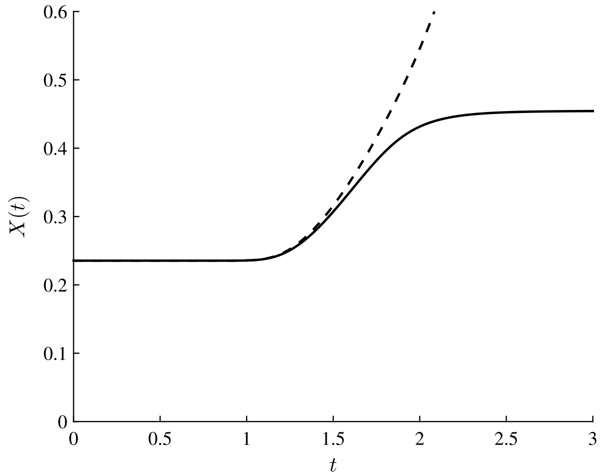

Fig. 4. Closed-loop response of the density at the bottleneck area under the predictor-feedback law (55) (solid line) and under the nominal, uncompensated feedback law (dashed line).

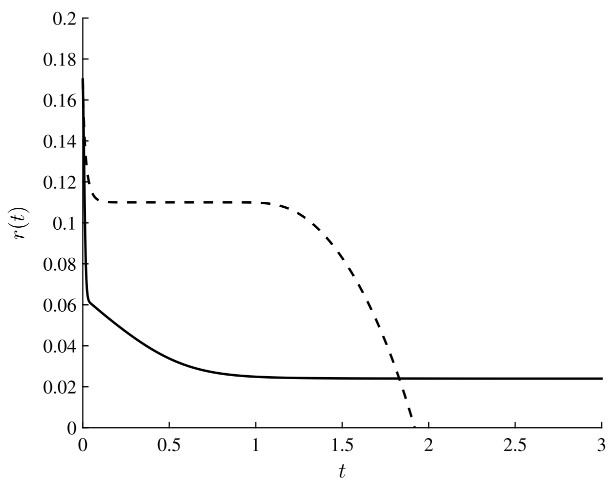

Fig. 5. Ramp flow (54), (55) (solid line) and the corresponding ramp flow of the nominal, uncompensated feedback law (dashed line).

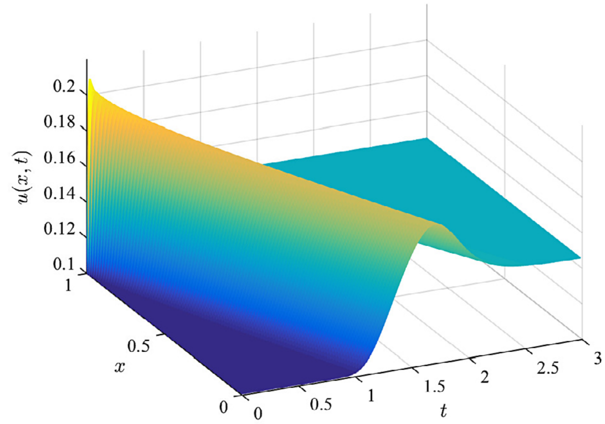

Fig. 6. Closed-loop response of the density at the highway stretch under the predictor-feedback law (55).

the density of the bottleneck area for which the maximum flow is achieved. In Fig. 4 we show the response of the density at the bottleneck area for the case of the predictor-feedback control law and for the case in which the uncompensated, nominal feedback law is applied. From Fig. 4 it is evident that the predictor-feedback law regulates the density at the desired equilibrium point. In contrast, when the uncompensated, nominal feedback law is applied, the density of the bottleneck area exceeds the critical density 0.5 and the nominal controller fails to stabilize the system. In Fig. 5 we show the ramp flow under the predictor-feedback law and under the nominal, uncompensated feedback law. Fig. 6 shows the response of the density at the highway stretch under the predictor-feedback controller.

6. Conclusions and future work

We presented a predictor-feedback control design methodology for nonlinear systems with actuator dynamics governed by quasilinear, first-order hyperbolic PDEs. We proved that the closed-loop system, under the developed feedback law, is locally asymptotically stable, utilizing Lyapunov-like arguments and ISS estimates. We also emphasized the relation of the considered PDE–ODE cascade to a system with input delay that depends on past input values. Finally, we presented a simulation example of control of a system that models the dynamics of vehicular traffic flow, taking into account the traffic flow dynamics at distant bottlenecks.

In the present paper, local stability of the closed-loop system is established despite the fact that the nominal plant is assumed to be ISS as well as forward complete and despite the positivity assumption of the transport speed. As it is also suggested in the simulation results of Section 5, it may be possible to obtain stability results under a local stabilizability assumption of the nominal plant and under the assumption that the transport speed is positive at the equilibrium point. Although establishing such a result would be far from trivial, the results in Bekiaris-Liberis and Krstic (2013b) (Section 6) may constitute a starting point.

A topic of future research may be the problem of boundary stabilization of general, quasilinear systems of first-order hyperbolic PDEs coupled with nonlinear ODE systems, as it is done in Di Meglio, Bribiesca Argomedo, Hu, and Krstic (2018) for the case in which both the PDE and ODE parts of the system are linear.

Appendix

Proof of Lemma 1

We first show that

$$p_t(x,t) = v\left(u(x,t)\right) p_x(x,t). \tag{A.1}$$

Differentiating (6) with respect to t and using (1), (2) as well as the fact that $p(0,t) = X(t)$, which immediately follows from (6) with $x = 0$, we get that

$$\begin{aligned} p_t(x,t) = {} & f\left(p(0,t), u(0,t)\right) + \int_0^x \frac{\partial f\left(p(y,t), u(y,t)\right)}{\partial p} \\ & \times p_t(y,t) \Gamma\left(u(y,t), u_y(y,t), y\right) dy \\ & + \int_0^x \frac{\partial f\left(p(y,t), u(y,t)\right)}{\partial u} v\left(u(y,t)\right) u_y(y,t) \\ & \times \Gamma\left(u(y,t), u_y(y,t), y\right) dy \\ & + \int_0^x f\left(p(y,t), u(y,t)\right) \Gamma_u\left(u(y,t), u_y(y,t), y\right) \\ & \times v\left(u(y,t)\right) u_y(y,t) dy + \int_0^x f\left(p(y,t), u(y,t)\right) \\ & \times \Gamma_{u_y}\left(u(y,t), u_y(y,t), y\right) \left(v'\left(u(y,t)\right) u_y(y,t)^2 \right. \\ & \left. + v\left(u(y,t)\right) u_{yy}(y,t)\right) dy. \end{aligned} \tag{A.2}$$

Differentiating (6) with respect to x we get that

$$v\left(u(x,t)\right)p_x(x,t) = \int_0^x \frac{\partial \Lambda(y,t)}{\partial y}dy + v\left(u(0,t)\right) \times f\left(p(0,t),u(0,t)\right) \times \Gamma\left(u(0,t),u_y(0,t),0\right) \tag{A.3}$$

$$\Lambda(y,t) = v\left(u(y,t)\right)f\left(p(y,t),u(y,t)\right) \times \Gamma\left(u(y,t),u_y(y,t),y\right), \tag{A.4}$$

and hence,

$$\begin{aligned} v\left(u(x,t)\right)p_x(x,t) = {} & v\left(u(0,t)\right)f\left(p(0,t),u(0,t)\right) \\ & \times \Gamma\left(u(0,t),u_y(0,t),0\right) \\ & + \int_0^x \frac{\partial f\left(p(y,t),u(y,t)\right)}{\partial p} \\ & \times p_y(y,t)v\left(u(y,t)\right) \\ & \times \Gamma\left(u(y,t),u_y(y,t),y\right)dy \\ & + \int_0^x \frac{\partial f\left(p(y,t),u(y,t)\right)}{\partial u} \\ & \times u_y(y,t)v\left(u(y,t)\right) \\ & \times \Gamma\left(u(y,t),u_y(y,t),y\right)dy \\ & + \int_0^x f\left(p(y,t),u(y,t)\right) \\ & \times \Gamma_u\left(u(y,t),u_y(y,t),y\right) \\ & \times v\left(u(y,t)\right)u_y(y,t)dy \\ & + \int_0^x f\left(p(y,t),u(y,t)\right) \\ & \times \Gamma_{u_y}\left(u(y,t),u_y(y,t),y\right) \\ & \times v\left(u(y,t)\right)u_{yy}(y,t)dy \\ & + \int_0^x f\left(p(y,t),u(y,t)\right) \\ & \times \Gamma_y\left(u(y,t),u_y(y,t),y\right) \\ & \times v\left(u(y,t)\right)dy \\ & + \int_0^x f\left(p(y,t),u(y,t)\right) \\ & \times \Gamma\left(u(y,t),u_y(y,t),y\right) \\ & \times v'\left(u(y,t)\right)u_y(y,t)dy. \end{aligned} \tag{A.5}$$

Comparing (A.2) with (A.5) and using the fact that $\Gamma\left(u(0,t),u_y(0,t),0\right) = \frac{1}{v(u(0,t))}$, which follows from (7) for $y = 0$, we arrive at

$$\begin{aligned} P(x,t) = {} & \int_0^x \frac{\partial f\left(p(y,t),u(y,t)\right)}{\partial p}P(y,t) \\ & \times \Gamma\left(u(y,t),u_y(y,t),y\right)dy \\ & + \int_0^x f\left(p(y,t),u(y,t)\right) \\ & \times \Gamma_{u_y}\left(u(y,t),u_y(y,t),y\right) \\ & \times v'\left(u(y,t)\right)u_y(y,t)^2dy \\ & - \int_0^x f\left(p(y,t),u(y,t)\right) \\ & \times \Gamma_y\left(u(y,t),u_y(y,t),y\right) \\ & \times v\left(u(y,t)\right)dy \\ & - \int_0^x f\left(p(y,t),u(y,t)\right) \\ & \times \Gamma\left(u(y,t),u_y(y,t),y\right) \\ & \times v'\left(u(y,t)\right)u_y(y,t)dy, \end{aligned} \tag{A.6}$$

where we defined

$$P(x,t) = p_t(x,t) - v\left(u(x,t)\right)p_x(x,t). \tag{A.7}$$

Using the definition of Γ in (7) we get that

$$\Gamma_{u_y}\left(u(y,t),u_y(y,t),y\right) = -\frac{yv'\left(u(y,t)\right)}{v(u(y,t))^2} \tag{A.8}$$

$$\Gamma_y\left(u(y,t),u_y(y,t),y\right) = -\frac{v'\left(u(y,t)\right)u_y(y,t)}{v(u(y,t))^2}. \tag{A.9}$$

Combining (A.8), (A.9) and using (7) we arrive at

$$\Gamma_{u_y}\left(u,u_y,y\right)v'(u)u_y^2 = \Gamma_y\left(u,u_y,y\right)v(u) - \frac{yv'(u)^2u_y^2}{v(u)^2} + \frac{v'(u)u_y}{v(u)} \tag{A.10}$$

$$\Gamma\left(u,u_y,y\right)v'(u)u_y = \frac{v'(u)u_y}{v(u)} - \frac{yv'(u)^2u_y^2}{v(u)^2}. \tag{A.11}$$

Since the right-hand sides of (A.10), (A.11) are equal, from (A.6) it follows that

$$P(x,t) = \int_0^x \frac{\partial f\left(p(y,t),u(y,t)\right)}{\partial p}P(y,t) \times \Gamma\left(u(y,t),u_y(y,t),y\right)dy. \tag{A.12}$$

Therefore, for each $t \geq 0$, the function P satisfies for all $x \in [0,1]$

$$P_x(x,t) = \frac{\partial f\left(p(x,t),u(x,t)\right)}{\partial p} \times \Gamma\left(u(x,t),u_x(x,t),x\right)P(x,t) \tag{A.13}$$

$$P(0,t) = 0. \tag{A.14}$$

Hence,

$$P \equiv 0, \tag{A.15}$$

which proves that indeed (A.1) holds. Therefore, differentiating (32) with respect to t we get that

$$w_t(x,t) = u_t(x,t) - \frac{\partial\kappa\left(p(x,t)\right)}{\partial p}v\left(u(x,t)\right) \times p_x(x,t). \tag{A.16}$$

Differentiating (32) with respect to x we get that

$$v\left(u(x,t)\right)w_x(x,t) = v\left(u(x,t)\right)u_x(x,t) - \frac{\partial\kappa\left(p(x,t)\right)}{\partial p}v\left(u(x,t)\right) \times p_x(x,t). \tag{A.17}$$

Combining (A.16) with (A.17) and using (2) we arrive at (33). Furthermore, since from (6) it holds that $p(0,t) = X(t)$, relation (35) follows from (1) and (32) for $x = 0$. Finally, relation (34) follows from (32) for $x = 1$ and (5), (3).

Proof of Lemma 2

Consider the following Lyapunov functional

$$L_{c,m}(t) = \int_0^1 e^{2(c+\lambda)xm}w(x,t)^{2m}dx + \int_0^1 e^{2(c+\lambda)xm}w_x(x,t)^{2m}dx, \tag{A.18}$$

for any $c > 0$ and any positive integer m, where (under Assumption 1)

$$w_{xt}(x,t) = v'\left(u(x,t)\right)u_x(x,t)w_x(x,t) + v\left(u(x,t)\right)w_{xx}(x,t) \tag{A.19}$$

$$w_x(1,t) = 0. \tag{A.20}$$

Under Assumption 1 (positivity of v), taking the time derivative of (A.18) along the solutions of (33)–(35), (A.19), (A.20) we get using integration by parts that

$$\begin{aligned}\dot{L}_{c,m}(t) \le & -\int_0^1 e^{2(c+\lambda)xm} w(x,t)^{2m} \\ & \times \left(2m(c+\lambda)v\left(u(x,t)\right) + v'\left(u(x,t)\right)u_x(x,t)\right)dx \\ & -\int_0^1 e^{2(c+\lambda)xm} w_x(x,t)^{2m} \\ & \times \left(2m(c+\lambda)v\left(u(x,t)\right) + v'\left(u(x,t)\right)u_x(x,t)\right. \\ & \left. -2mv'\left(u(x,t)\right)u_x(x,t)\right)dx. \end{aligned} \tag{A.21}$$

Using (8) and the fact that $m \ge 1$ we obtain for all $x \in [0,1]$ and $t \ge 0$

$$\begin{aligned}2m\left(-\lambda+M\right)v\left(u(x,t)\right) \ge & -2m\lambda v\left(u(x,t)\right) \\ & - v'\left(u(x,t)\right)u_x(x,t)\end{aligned} \tag{A.22}$$

$$\begin{aligned}2m\left(-\lambda+1\right)v\left(u(x,t)\right) \ge & -2m\lambda v\left(u(x,t)\right) - (1-2m) \\ & \times v'\left(u(x,t)\right)u_x(x,t).\end{aligned} \tag{A.23}$$

Therefore, choosing any λ such that $\lambda \ge 1+M$, it follows from (A.21) that

$$\begin{aligned}\dot{L}_{c,m}(t) \le & -2mc\underline{v}\int_0^1 e^{2(c+\lambda)xm} w(x,t)^{2m} \\ & -2mc\underline{v}\int_0^1 e^{2(c+\lambda)xm} w_x(z,t)^{2m}dx,\end{aligned} \tag{A.24}$$

where we also used (4). Thus,[4]

$$\dot{L}_{c,m}(t) \le -2mc\underline{v}L_{c,m}(t), \tag{A.25}$$

which implies that

$$L_{c,m}^{\frac{1}{2m}}(t) \le e^{-c\underline{v}(t-s)}L_{c,m}^{\frac{1}{2m}}(s), \quad \text{for all } t \ge s \ge 0. \tag{A.26}$$

Moreover, from (A.18) it follows that

$$\Xi_{c,m}(t) \le 2e^{-c\underline{v}(t-s)}\Xi_{c,m}(s), \quad \text{for all } t \ge s \ge 0, \tag{A.27}$$

where

$$\begin{aligned}\Xi_{c,m}(t) = & \left(\int_0^1 e^{2(c+\lambda)xm} w(x,t)^{2m}dx\right)^{\frac{1}{2m}} \\ & + \left(\int_0^1 e^{2(c+\lambda)xm} w_x(x,t)^{2m}dx\right)^{\frac{1}{2m}}.\end{aligned} \tag{A.28}$$

Taking the limit of (A.27) as m goes to infinity we obtain

$$\Xi_c(t) \le 2e^{-c\underline{v}(t-s)}\Xi_c(s), \quad \text{for all } t \ge s \ge 0, \tag{A.29}$$

where

$$\begin{aligned}\Xi_c(t) = & \max_{0\le x\le 1}\left|e^{x(c+\lambda)}w(x,t)\right| \\ & + \max_{0\le x\le 1}\left|e^{x(c+\lambda)}w_x(x,t)\right|,\end{aligned} \tag{A.30}$$

which follows from definition (A.28) using the fact that $\lim_{m\to\infty}\Xi_{c,m}(t) = \Xi_c(t)$. It follows, for all $t \ge s \ge 0$, that

$$\begin{aligned}\|w(t)\|_C + \|w_x(t)\|_C \le & \, 2e^{-c\underline{v}(t-s)}e^{(c+\lambda)} \\ & \times \left(\|w(s)\|_C + \|w_x(s)\|_C\right).\end{aligned} \tag{A.31}$$

[4] Note that although the estimate (A.25) for $L_{c,m}$ is derived for u that is of class C^2 (and thus, so is w satisfying (A.19), (A.20)), the estimate (A.25) remains valid (in the distribution sense) when u is only of class C^1, see, e.g., Coron & Bastin (2016).

Under Assumption 3 (see, e.g., Sontag & Wang, 1995) we obtain from (35) that

$$|X(t)| \le \beta_1\left(|X(s)|, t-s\right) + \gamma_1\left(\sup_{s\le\tau\le t}|w(0,\tau)|\right), \tag{A.32}$$

for all $t \ge s \ge 0$, some class $\mathcal{KL}$ function β_1, and some class $\mathcal{K}$ function γ_1. Mimicking the arguments in the proof of Lemma 4.7 from Khalil (2002), we set $s = \frac{t}{2}$ in (A.32) to get that

$$\begin{aligned}|X(t)| \le & \, \beta_1\left(\left|X\left(\frac{t}{2}\right)\right|, \frac{t}{2}\right) \\ & + \gamma_1\left(\sup_{\frac{t}{2}\le\tau\le t}\|w(\tau)\|_C\right),\end{aligned} \tag{A.33}$$

and thus, using (A.32) for $s = 0$ and $t \to \frac{t}{2}$ we arrive at

$$\begin{aligned}|X(t)| \le & \, \beta_1\left(\beta_1\left(|X(0)|, \frac{t}{2}\right) + \gamma_1\left(\sup_{0\le\tau\le\frac{t}{2}}\|w(\tau)\|_C\right), \frac{t}{2}\right) \\ & + \gamma_1\left(\sup_{\frac{t}{2}\le\tau\le t}\|w(\tau)\|_C\right), \quad \text{for all } t \ge 0.\end{aligned} \tag{A.34}$$

Moreover, using (A.31) we get that

$$\sup_{0\le\tau\le\frac{t}{2}}\|w(\tau)\|_C \le 2e^{(c+\lambda)}\left(\|w(0)\|_C + \|w_x(0)\|_C\right) \tag{A.35}$$

$$\begin{aligned}\sup_{\frac{t}{2}\le\tau\le t}\|w(\tau)\|_C \le & \, 2e^{-c\underline{v}\frac{t}{2}}e^{(c+\lambda)}\left(\|w(0)\|_C\right. \\ & \left. + \|w_x(0)\|_C\right).\end{aligned} \tag{A.36}$$

Therefore, combining (A.34) with (A.35), (A.36) and using (A.31) we get (36) with

$$\begin{aligned}\beta_w(s,t) = & \, \beta_1\left(\beta_1(s,0) + \gamma_1\left(2e^{(c+\lambda)}s\right), \frac{t}{2}\right) \\ & + 2e^{-c\underline{v}t}e^{(c+\lambda)}s + \gamma_1\left(2e^{-c\underline{v}\frac{t}{2}}e^{(c+\lambda)}s\right).\end{aligned} \tag{A.37}$$

Proof of Lemma 3

Differentiating relation (6) with respect to x we get that, for each t, p satisfies the following ODE in x

$$p_x(x,t) = f\left(p(x,t), u(x,t)\right)\Gamma\left(u(x,t), u_x(x,t), x\right) \tag{A.38}$$

$$p(0,t) = X(t). \tag{A.39}$$

Under Assumption 2 there exists a smooth function $R : \mathbb{R}^n \to \mathbb{R}_+$ and class $\mathcal{K}_\infty$ functions α_1, α_2, and α_3 such that (see, e.g., Angeli & Sontag, 1999; Krstic, 2009, 2010)

$$\alpha_1\left(|X|\right) \le R(X) \le \alpha_2\left(|X|\right) \tag{A.40}$$

$$\frac{\partial R(X)}{\partial X}f(X,\omega) \le R(X) + \alpha_3\left(|\omega|\right), \tag{A.41}$$

for all $(X,\omega)^{\mathrm{T}} \in \mathbb{R}^{n+1}$. From relation (8) it follows that

$$\begin{aligned}v\left(u(x,t)\right) - xv'\left(u(x,t)\right)u_x(x,t) > 0, \quad & \text{for all } x \in [0,1] \\ & \text{and } t \ge 0,\end{aligned} \tag{A.42}$$

which can be seen considering separately the cases $v'\left(u(x,t)\right)u_x(x,t) \le 0$ and $v'\left(u(x,t)\right)u_x(x,t) > 0$, and using (4). Hence, from the definition of Γ in (7) and (4) we conclude that

$$\Gamma\left(u(x,t), u_x(x,t), x\right) > 0, \quad \text{for all } x \in [0,1] \text{ and } t \ge 0. \tag{A.43}$$

Thus, from (A.41) it follows that

$$\frac{\partial R\left(p(x,t)\right)}{\partial p} f\left(p(x,t), u(x,t)\right) \Gamma\left(u(x,t), u_x(x,t), x\right) \leq \Gamma\left(u(x,t), u_x(x,t), x\right)\left(R\left(p(x,t)\right) + \alpha_3\left(|u(x,t)|\right)\right). \tag{A.44}$$

From (7), using relations (4) and (8) it also follows that

$$\Gamma\left(u(x,t), u_x(x,t), x\right) \leq \frac{2+M}{\underline{v}}, \quad \text{for all } x \in [0,1] \text{ and } t \geq 0, \tag{A.45}$$

and thus, from (A.44) we get using (A.38) that

$$\frac{\partial R\left(p(x,t)\right)}{\partial x} \leq \frac{2+M}{\underline{v}}\left(R\left(p(x,t)\right) + \alpha_3\left(|u(x,t)|\right)\right). \tag{A.46}$$

Employing the comparison principle and using (A.39) we arrive at

$$R\left(p(x,t)\right) \leq e^{\frac{2+M}{\underline{v}}x} R\left(X(t)\right) + \frac{2+M}{\underline{v}} \times \int_0^x e^{\frac{2+M}{\underline{v}}(x-y)} \alpha_3\left(|u(y,t)|\right) dy, \tag{A.47}$$

for all $x \in [0,1]$ and $t \geq 0$. Hence, using (A.40) we get

$$\|p(t)\|_C \leq \alpha_4\left(|X(t)| + \|u(t)\|_C\right), \tag{A.48}$$

where

$$\alpha_4(s) = \alpha_1^{-1}\left(e^{\frac{2+M}{\underline{v}}}\left(\alpha_2(s) + \alpha_3(s)\right)\right). \tag{A.49}$$

Since f is continuously differentiable with $f(0,0) = 0$ we conclude that there exists a class $\mathcal{K}_\infty$ function α_5 such that

$$|f(X,\omega)| \leq \alpha_5\left(|X| + |\omega|\right). \tag{A.50}$$

Thus, using (A.45) and (A.48), we get from (A.38)

$$|p_x(x,t)| \leq \alpha_6\left(|X(t)| + \|u(t)\|_C\right), \tag{A.51}$$

where

$$\alpha_6(s) = \frac{2+M}{\underline{v}} \alpha_5\left(\alpha_4(s) + s\right). \tag{A.52}$$

The proof is completed by taking the maximum, with respect to $x \in [0,1]$, in both sides of (A.51) and setting $\rho_1(s) = \alpha_4(s) + \alpha_6(s)$.

Proof of Lemma 4

We start defining the change of variables with respect to x

$$z(x,t) = \frac{x}{v\left(u(x,t)\right)}, \tag{A.53}$$

which is well-defined thanks to (4) and where t acts as a parameter. Using the fact that

$$\Gamma\left(u(x,t), u_x(x,t), x\right) = \frac{\partial z(x,t)}{\partial x}, \tag{A.54}$$

it follows from (A.43) that the function z is strictly increasing with respect to x, for each t. Thus, it admits an inverse defined for each t as $x = \chi(z,t)$. Therefore, from relations (A.38), (A.39), and definition (A.54) we obtain

$$\bar{p}_z(z,t) = f\left(\bar{p}(z,t), \bar{u}(z,t)\right), \quad z \in \left[0, \frac{1}{v\left(u(1,t)\right)}\right] \tag{A.55}$$

$$\bar{p}(0,t) = X(t), \tag{A.56}$$

where

$$\bar{p}(z,t) = p\left(\chi(z,t), t\right) \tag{A.57}$$

$$\bar{u}(z,t) = u\left(\chi(z,t), t\right). \tag{A.58}$$

Moreover, setting $x = \chi(z,t)$ in relation (32) we get that

$$\bar{u}(z,t) = \kappa\left(\bar{p}(z,t)\right) + \bar{w}(z,t), \tag{A.59}$$

where

$$\bar{w}(z,t) = w\left(\chi(z,t), t\right). \tag{A.60}$$

Thus, we re-write (A.55) as

$$\bar{p}_z(z,t) = f\left(\bar{p}(z,t), \kappa\left(\bar{p}(z,t)\right) + \bar{w}(z,t)\right), \quad z \in \left[0, \frac{1}{v\left(u(1,t)\right)}\right]. \tag{A.61}$$

Under Assumption 3 there exist a smooth function $S : \mathbb{R}^n \to \mathbb{R}_+$ and class $\mathcal{K}_\infty$ functions $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3$, and $\hat{\alpha}_4$ such that (see, e.g., Sontag & Wang, 1995)

$$\hat{\alpha}_1\left(|X|\right) \leq S(X) \leq \hat{\alpha}_2\left(|X|\right) \tag{A.62}$$

$$\frac{\partial S(X)}{\partial X} f\left(X, \kappa(X) + \omega\right) \leq -\hat{\alpha}_3\left(|X|\right) + \hat{\alpha}_4\left(|\omega|\right). \tag{A.63}$$

Thus, we get from (A.63) that

$$\frac{\partial S\left(\bar{p}(z,t)\right)}{\partial \bar{p}} f\left(\bar{p}(z,t), \kappa\left(\bar{p}(z,t)\right) + \bar{w}(z,t)\right) \leq -\hat{\alpha}_3\left(|\bar{p}(z,t)|\right) + \hat{\alpha}_4\left(|\bar{w}(z,t)|\right), \tag{A.64}$$

and hence, using (A.61), (A.56) and integrating from 0 to z we obtain

$$S\left(\bar{p}(z,t)\right) \leq S\left(X(t)\right) + \int_0^z \hat{\alpha}_4\left(|\bar{w}(y,t)|\right) dy, \quad z \in \left[0, \frac{1}{v\left(u(1,t)\right)}\right]. \tag{A.65}$$

Using (4) and (A.62) we get from (A.65) that

$$|\bar{p}(z,t)| \leq \hat{\alpha}_1^{-1}\left(\hat{\alpha}_2\left(|X(t)|\right) + \frac{1}{\underline{v}}\hat{\alpha}_4\left(\max_{0 \leq z \leq \frac{1}{v(u(1,t))}} |\bar{w}(z,t)|\right)\right), \quad z \in \left[0, \frac{1}{v\left(u(1,t)\right)}\right]. \tag{A.66}$$

Taking a maximum of both sides in (A.66), with definitions (A.57), (A.58), and (A.60) we arrive at

$$\|p(t)\|_C \leq \hat{\alpha}_5\left(|X(t)| + \|w(t)\|_C\right), \tag{A.67}$$

with $\hat{\alpha}_5(s) = \hat{\alpha}_1^{-1}\left(\hat{\alpha}_2(s) + \frac{1}{\underline{v}}\hat{\alpha}_4(s)\right)$. Under Assumption 3 (continuity of κ and the fact that $\kappa(0) = 0$) there exists a class $\mathcal{K}_\infty$ function $\hat{\hat{\alpha}}_1$ such that

$$|\kappa(X)| \leq \hat{\hat{\alpha}}_1\left(|X|\right). \tag{A.68}$$

Therefore, using (A.50) we get from (A.61) that

$$|\bar{p}_z(z,t)| \leq \alpha_5\left(|\bar{p}(z,t)| + \hat{\hat{\alpha}}_1\left(|\bar{p}(z,t)|\right) + |\bar{w}(z,t)|\right). \tag{A.69}$$

Using (A.66) we arrive at

$$|\bar{p}_z(z,t)| \leq \hat{\alpha}_6\left(|X(t)| + \max_{0 \leq z \leq \frac{1}{v(u(1,t))}} |\bar{w}(z,t)|\right), \tag{A.70}$$

where $\hat{\alpha}_6(s) = \alpha_5\left(\hat{\alpha}_5(s) + \hat{\hat{\alpha}}_1\left(\hat{\alpha}_5(s)\right) + s\right)$. Using definition (A.57), from relations (A.45), (A.54) we obtain that

$$|p_x(x,t)| \leq \frac{2+M}{\underline{v}} |\bar{p}_z(z,t)|, \tag{A.71}$$

and hence, from (A.70) we arrive at

$$\|p_x(t)\|_C \leq \frac{2+M}{\underline{v}}\hat{\alpha}_6\left(|X(t)| + \max_{0\leq z\leq \frac{1}{v(u(1,t))}} |\bar{w}(z,t)|\right). \tag{A.72}$$

With definition (A.60) we get that

$$\|p_x(t)\|_C \leq \frac{2+M}{\underline{v}}\hat{\alpha}_6\left(|X(t)| + \|w(t)\|_C\right), \tag{A.73}$$

and hence, the lemma is proved with $\rho_2(s) = \hat{\alpha}_5(s) + \frac{2+M}{\underline{v}}\hat{\alpha}_6(s)$.

Proof of Lemma 5

Under Assumption 3 (continuous differentiability of κ) there exists a class $\mathcal{K}_\infty$ function $\hat{\hat{\alpha}}_2$ such that

$$|\nabla\kappa(X)| \leq |\nabla\kappa(0)| + \hat{\hat{\alpha}}_2(|X|), \tag{A.74}$$

for all $X \in \mathbb{R}^n$. Therefore, using (38) and (A.68), we get from (32) that

$$\begin{aligned}|w(x,t)| + |w_x(x,t)| \leq{}& |u(x,t)| + |u_x(x,t)| \\ &+ \hat{\hat{\alpha}}_1\left(\rho_1\left(|X(t)| + \|u(t)\|_C\right)\right) \\ &+ \Big(|\nabla\kappa(0)| + \hat{\hat{\alpha}}_2\big(\rho_1(|X(t)| \\ &+ \|u(t)\|_C)\big)\Big) \\ &\times \rho_1\left(|X(t)| + \|u(t)\|_C\right),\end{aligned} \tag{A.75}$$

and thus, taking a maximum in both sides of (A.75), estimate (40) follows with

$$\rho_3(s) = s + \hat{\hat{\alpha}}_1(\rho_1(s)) + \left(|\nabla\kappa(0)| + \hat{\hat{\alpha}}_2(\rho_1(s))\right) \times \rho_1(s). \tag{A.76}$$

Similarly, using (A.68), (A.74), and (39) we get estimate (41) with

$$\rho_4(s) = s + \hat{\hat{\alpha}}_1(\rho_2(s)) + \left(|\nabla\kappa(0)| + \hat{\hat{\alpha}}_2(\rho_2(s))\right) \times \rho_2(s). \tag{A.77}$$

Proof of Lemma 6

Under Assumption 1 (continuous differentiability of v) we conclude that there exists a class $\mathcal{K}_\infty$ function $\hat{\rho}$ such that

$$\left|v'(u(x,t))\right| \leq \left|v'(0)\right| + \hat{\rho}(|u(x,t)|), \tag{A.78}$$

and hence, for all $x \in [0, 1]$ and $t \geq 0$ it holds that

$$\left|v'(u(x,t))\right| \leq \left|v'(0)\right| + \hat{\rho}(\|u(t)\|_C). \tag{A.79}$$

Thus, it holds that

$$\left|v'(u(x,t))\,u_x(x,t)\right| \leq \left(\left|v'(0)\right| + \hat{\rho}(\|u(t)\|_C)\right) \times \|u_x(t)\|_C, \tag{A.80}$$

for all $x \in [0, 1]$ and $t \geq 0$. From relation (4), one can conclude that whenever

$$\left|v'(u(x,t))\,u_x(x,t)\right| \leq \epsilon, \tag{A.81}$$

where ϵ is any constant such that $0 < \epsilon < \underline{v}$, relation (8) holds with any M such that $M > \frac{\epsilon}{\underline{v}}$. Consequently, choosing any constant δ_1 such that

$$\delta_1 \leq \psi^{-1}(\epsilon), \tag{A.82}$$

where

$$\psi(s) = \left(\left|v'(0)\right| + \hat{\rho}(s)\right)s, \tag{A.83}$$

completes the proof.

Proof of Lemma 7

Combining estimate (41) with (36) we obtain

$$\Omega(t) \leq \rho_4\left(\beta_w\left(\Omega_w(0), t\right)\right), \tag{A.84}$$

and hence, with (40) and the properties of class $\mathcal{KL}$ functions we arrive at

$$\Omega(t) \leq \rho_4\left(\beta_w\left(\rho_3\left(\Omega(0)\right), 0\right)\right). \tag{A.85}$$

Therefore, for all initial conditions that satisfy the bound (27) with any δ such that

$$\delta \leq \phi^{-1}(\delta_1), \tag{A.86}$$

where

$$\phi(s) = \rho_4\left(\beta_w\left(\rho_3(s, 0)\right)\right), \tag{A.87}$$

the solutions satisfy (42). Furthermore, from Lemma 6, it follows that for all of those initial conditions, the solutions verify (8).

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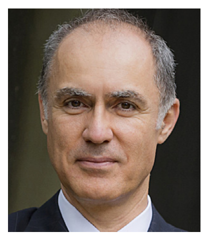

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