Nonlinear Energy Harvesting Models in Wireless Information and Power Transfer

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Abstract—This work compares different linear and nonlinear RF energy harvesting models, including limited or unlimited sensitivity, for simultaneous wireless information and power transfer (SWIPT). The probability of successful SWIPT reception under a family of RF harvesting models is rigorously quantified, using state-of-the-art rectifiers in the context of commercial RFIDs. A significant portion of SWIPT literature uses oversimplified models that do not account for limited sensitivity or nonlinearity of the underlying harvesting circuitry. This work demonstrates that communications signals are not always appropriate for simultaneous energy transfer and concludes that for practical SWIPT studies, the inherent non-ideal characteristics of the harvester should be carefully taken into account; specific harvester’s modeling methodology is also offered.

I. INTRODUCTION

Intense research has been devoted the last years on simultaneous wireless information and power transfer (SWIPT). The main concept in far field SWIPT systems is the exploitation of the communication signals for radio frequency (RF) energy harvesting, typically with rectennas, i.e., antenna and rectifier(s). The latter perform the required RF-to-DC conversion, including one (or more) diode(s). The main problem in far field RF energy harvesting is the limited sensitivity of the circuit, currently in the order of $-35 \text{ dBm}$ to $-25 \text{ dBm}$, with slow improvement by a factor of 2 every approximately 5 years [7]. Such power levels below which energy transfer cannot be performed, are orders of magnitude higher than current communications circuits sensitivity, which may reach values as low as $-130 \text{ dBm}$ to $-80 \text{ dBm}$, depending on bandwidth. Thus, signals appropriate for communications may not be simultaneously suitable for energy transfer [7], [8].

Another major issue in the SWIPT literature is the adoption of oversimplified RF harvesting models, which either exhibit a linear relationship between input RF and output harvested power or assume unlimited sensitivity. Rectennas, due to the presence of diode(s), exhibit a highly nonlinear behavior, with limited sensitivity, due to the need for bias. Despite the vast amount of literature in the wireless communications theory community that adheres to the above assumptions, exceptions have only recently started to emerge; for example, work in [7], [8] utilized convex optimization techniques to optimize the parameters of multi-tone waveforms, which improve RF harvesting efficiency compared to single-tone, while taking into account the nonlinearity of the rectifier. Other nonlinear RF harvesting models have been recently proposed, which however miss the limited sensitivity issue and will be discussed subsequently.

Therefore, there is a strong need to evaluate different RF harvesting models, taking into account both harvesting sensitivity and nonlinearity, as well as facts from the relevant microwave literature. Radio frequency identification (RFID) technology is the most prominent example of SWIPT, with significant prior art, as well as commercial interest. This work compares different linear and nonlinear energy harvesting models for SWIPT, taking also into account limited or unlimited sensitivity; comparisons are performed based on real, state-of-the-art rectifiers [7] in RFID, using backscatter communications. It is found that neglecting harvester’s nonlinearity and limited sensitivity may offer misleading results.

II. SIGNAL MODEL

Backscatter radio/RFID technology is the most prominent example of SWIPT. A monostatic, single-antenna reader topology is examined with reader and tag, depicted in Fig. 1. In that case, the illuminating carrier emitter and the receiver of the tag-backscattered signal is the same, full-duplex unit, a.k.a. the reader; the latter is equipped with a single antenna serving both reception and transmission, using an appropriate duplexer, the circulator. Thus, path-loss and small-scale fading are the same for both reader-to-tag (downlink) and tag-to-reader (uplink) links. Both links are subject to large-scale fading, where the path-gain at tag-to-reader distance $d$ is given by:

$$L \equiv L(d) = \left( \frac{\lambda}{4\pi d_0} \right)^2 \left( \frac{d_0}{d} \right)^\nu,$$  \hspace{1cm} (1)

where $d_0$ is a reference distance (assumed unit thereinafter), $\lambda$ is the wavelength and $\nu$ is the path loss exponent.

Flat fading is assumed due to relatively small communication bandwidth. Thus, small-scale fading coefficient, for both downlink and uplink is given by $h = ae^{-j\phi}$. Due to potential strong line-of-sight (LoS), Nakagami small-scale fading is assumed with $\mathbb{E}[a^2] = 1$ and Nakagami parameter $\kappa \geq \frac{1}{2}$ [7, p. 79]. The special cases of Rayleigh fading and no fading ($\kappa = 1$) are obtained for $\kappa = 1$ and $\kappa = \infty$, respectively.

Assuming the reader emits an unmodulated carrier with transmit power $P_R$ and frequency $F_c$, the impinged signal at the tag signal can be expressed as follows:

$$c_T(t) = \sqrt{2LP_R} \Re\{h e^{j2\pi F_c t}\}.$$  \hspace{1cm} (2)
The received power at the tag is then given by:

$$P_{\text{in}} = L P_R |h|^2 = L P_R a^2.$$  \hspace{1cm} (3)

According to the above, $P_{\text{in}}$ follows Gamma distribution $\mathbb{E}(\alpha^2) = 1$: $f_{P_{\text{in}}}(x) = \left(\frac{\alpha}{LP_R}\right)^x \frac{e^{-\frac{\alpha}{LP_R}x}}{\Gamma(x)}$, $x \geq 0$, where $(\alpha, \frac{LP_R}{\alpha})$ are the shape and scale parameter, respectively, and $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t} dt$ is the Gamma function.

III. RFID TAG OPERATION

The RFID tag does not include any power-demanding signal conditioning units, e.g., amplifiers, mixers or oscillators (Fig. 1). Instead, communication is achieved by varying the reflection coefficient between tag antenna and its termination loads, using a RF switch. Binary modulation is achieved with two different reflection coefficients (i.e., two different termination loads $Z_0, Z_1$). This operation results to modulation of tag information on top of the reader illuminating signal, reflected (from the tag) back to the reader, in an ultra low-power fashion.

A. RF Harvesting & Tag Powering

In order for the RFID tag to operate, power must be harvested from the impinging, reader-generated signal. Input power must be above the tag harvester sensitivity $P_{\text{sen}}$, i.e., $P_{\text{in}} > P_{\text{sen}}$. $P_{\text{sen}}$ is a crucial parameter in backscatter communication with passive tags, due to the fact that state-of-the-art, far field RF harvesters offer limited sensitivity.

Work in [7] established that a high-order polynomial in the dBm scale can be safely considered as ground truth model for harvesting efficiency function; thus, harvested power can be modeled as a function of input power $x$ as follows:

$$p(x) = \begin{cases} 0, & x \in [0, P_{\text{sen}}) \\ \left(w_0 + \sum_{i=1}^{W} w_i (10 \log_{10}(x))^i\right) \cdot \eta, & x \in [P_{\text{sen}}, P_{\text{sat}}], \\ p(P_{\text{sat}}), & x \geq P_{\text{sat}}, \end{cases}$$  \hspace{1cm} (4)

where $\eta$ and $p(x)$ take values in mWatt, while the quantity $(w_0 + \sum_{i=1}^{W} w_i (10 \log_{10}(x))^i)$ is the harvesting efficiency function, with $W$ being the degree of the polynomial and $\{w_i\}_{i=0}^{W}$ the corresponding coefficients. For the analysis below we assume that function $p(x)$ is continuous and increasing in $[P_{\text{sen}}, P_{\text{sat}}]$. As shown in [7], the parameters $\{w_i\}_{i=0}^{W}$ in Eq. (4) can be obtained directly from harvesters’ data using standard convex optimization fitting methods.

Several models have been proposed in order for the harvested power to be mathematically described. These models are summarized below:

1) Linear Model (L): Single parameter model, where the harvested power can be expressed as $p_1(x) = \eta_1 x$, $x \geq 0$. This is the most utilized model in SWIPT literature, it’s linear and does not account for harvesters’ sensitivity.

2) Constant Linear (CL): Linear model with the addition of taking into account the sensitivity of the harvester. According to that model, harvested power is expressed as $p_2(x) = \eta_{\text{CL}} \cdot (x-P_{\text{sen}})$ for $x \in [P_{\text{sen}}, \infty)$ and zero in the rest of its domain; $\eta_{\text{CL}}$ is the constant harvesting efficiency.

3) Nonlinear Normalized Sigmoid: The model was proposed in [?] and assumes $P_{\text{sen}} = 0$, i.e., it does not account for harvesters’ sensitivity. The harvested power is expressed as:

$$p_3(x) = \frac{1+\exp(-a_1(x-b_1))}{1+\exp(a_0+b_0)}.$$  \hspace{1cm} (5)

The shape of $p_3(x)$ is determined by three real numbers $a_0, b_0$, and $a_1$. A similar, sigmoid model accounting however for $P_{\text{sen}}$, was proposed in [?], where the harvested power is modeled as:

$$p_4(x) = \max\left\{\frac{c_1}{\exp(-a_1 P_{\text{sen}} + b_1)} \left(\frac{1+\exp(-a_1 P_{\text{sen}} + b_1)}{1+\exp(-a_1 P_{\text{sen}} + b_1)} - 1\right) \cdot 0\right\}.$$  \hspace{1cm} (6)

4) Second Order Polynomial: In [?] a model based on a second degree polynomial in milliWatt domain has been suggested. Following that model, harvested power can be expressed as $p_5(x) = a_2 x^2 + b_2 x + c_2$. The above model does not account for $P_{\text{sen}}$. In order to encompass the effect of sensitivity, $p_5(\cdot)$ can be modified as

$$p_6(x) = a_3 \cdot (x-P_{\text{sen}})^2 + b_3 \cdot (x-P_{\text{sen}}).$$  \hspace{1cm} (6)

The parameters of the model in Eq. (6) are $a_3, b_3$ and $P_{\text{sen}}$.

5) Piecewise Linear Model: Given a set of $J+1$ data pairs of input power and corresponding harvested power, denoted as $\{(q_j)_{j=0}^J \}$ and $\{(v_j)_{j=0}^J \}$, respectively, slopes $l_j \triangleq \frac{q_{j+1} - q_j}{v_{j+1} - v_j}$, $j \in [J]$, are defined, where $[J] \triangleq \{1,2,\ldots,J\}$. Modeling sensitivity and saturation characteristics is done through points $q_0 = P_{\text{sen}}$ and $q_J = P_{\text{sat}}$. Having those slopes, the harvested power is given by:

$$p_7(x) = \begin{cases} 0, & x \in [0, q_0] \\ l_j (x - q_{j-1}) + v_{j-1}, & x \in (q_{j-1}, q_j], \forall j \in [J], \\ v_J, & x \in [q_J, \infty). \end{cases}$$  \hspace{1cm} (7)

Function $p_7(x)$ is defined using $2(J+1)$ real numbers, easily available from harvesters’ specifications; thus, determining $p_7(x)$ is straightforward, without any tuning.

It should be noted that the last model can potentially model energy harvesting from other sources, other than RF. For instance, if photodiodes are used in order to harvest energy from either ambient or solar light, the proposed model can
Eq. (4) adheres to the data; the rest of the nonlinear harvested energy, as ground truth the specification data; the nonlinear model incorporates the non-ideal factor of the diode, bias saturation current, (ii) the thermal voltage, and (iii) the hardware characteristics of the rectifier and the modeling truncation parameter. Note that the above model depends on changes for different type of RF harvesters.

The output DC current is expressed as:

\[
t_{\text{out}} = \sum_{i=0}^{n_0} \kappa_i R_{\text{a}}^{1/2} E_t[|s_T(t)|^2]
\]

where diode-dependent parameter \( \kappa_i \) depends on (i) the reverse bias saturation current, (ii) the thermal voltage, and (iii) the ideality factor of the diode, \( R_{\text{a}} \) is the impedance, and \( n_0 \) is a truncation parameter. Note that the above model depends on the hardware characteristics of the rectifier and the modeling changes for different type of RF harvesters.

Fig. 2 illustrates the harvested power (in mWatt) versus input power (in dBm) for the harvester proposed in [7] using nonlinear harvested power function \( \rho_n(s,.) \), \( n = 3, 4, 5, 6 \), as well as for the ground truth model in Eq. (4). Input power range within \([-45,-20] \) dBm.

![Harvested Power vs Input Power](image)

### 6) Diode-Based Hardware-Specific Model:
In [7], [8] a hardware-specific, nonlinear model is proposed. That model is based on the physics of the diode and links the output DC current (and therefore DC power) to the input signal (power and shape). For a specific RF energy harvesting circuit with one or multiple diode, the output DC current is expressed as:

\[
t_{\text{out}} = \sum_{i=0}^{n_0} \kappa_i R_{\text{a}}^{1/2} E_t[|s_T(t)|^2]
\]

where diode-dependent parameter \( \kappa_i \) depends on (i) the reverse bias saturation current, (ii) the thermal voltage, and (iii) the ideality factor of the diode, \( R_{\text{a}} \) is the impedance, and \( n_0 \) is a truncation parameter. Note that the above model depends on the hardware characteristics of the rectifier and the modeling changes for different type of RF harvesters.

During normal operation, tags’ antenna is terminated at load \( Z_0 \) (absorbing state, see Fig. 1) for a time fraction of \( \tau \) while for the rest \( 1 - \tau \) antenna is connected to \( Z_1 \) (reflection state). Given that the tag is at \( Z_0 \), a portion \( \chi \) of the received power is destined solely for energy harvesting, i.e., \( \xi_{\text{har}} = \chi \tau \) is \( (0,1) \) percentage of input power is dedicated for RF energy harvesting. The rest \((1-\chi)\tau\), is exploited for downlink communication purposes.

Thus, for the tag to operate, the total harvested power \( p_{\text{har}}(P_{\text{in}}) \) must be greater than the tag overall power consumption \( P_c \). This is critical, given the fact that batteryless RFID tags typically incorporate no energy storage element, e.g., (super)capacitor, due to size and cost limitations.

### B. Backscatter Communication

As stated earlier, the tag alters the load terminating its antenna using a switch. Load \( Z_0 \) is, by construction, designed to match antennas’ impedance. Thus, when antenna is terminated at \( Z_0 \), the load absorbs (ideally, if perfectly matched) all the power offered by the impinged signal. When the antenna is terminated at \( Z_1 \), a fraction \( \rho_n \leq 1 - \tau \) of the impinged power is used for uplink scatter radio operation. Parameter \( \rho_n \) depends on the tag scattering efficiency (which also incorporates non-idealities from the above model). Modified reflection coefficient \([7] \ \Gamma_i \), when the antenna is terminated at \( Z_i \), \( i \in \{0,1\} \), is given by \( \Gamma_i = \frac{Z_i - Z_a}{Z_i + Z_a} \), where \( Z_a \) antenna’s impedance. The baseband equivalent of the tag-backscattered signal can be expressed as \([7] \ A_s - \Gamma_i \), which in turn depends on the (load-independent) tag antenna structural mode \( A_s \) and the transmitted bit \( i \); the backscattered baseband signal, for a duration of \( N \) bit tags, is given by \([7] \):

\[
b(t) = \sqrt{\rho_n P_{\text{R}}} h \left( A_s - \Gamma_0 + \Delta \sum_{n=1}^{N} s_n(t - (n-1)T) \right),
\]

where, \( \Delta \Gamma \triangleq (\Gamma_0 - \Gamma_1) \), \( b_n \in \{0,1\} \) is the \( n \)-th reflected bit, while function \( s_n(\cdot) \) is the backscattered signal basis function, of duration \( T \), when bit \( b_n \) is transmitted.

In order to a) balance the time for which the tag is absorbing energy, independently of the tag’s data bits, and b) avoid ghost tag reception, i.e., reader misinterpreting thermal noise as tag information, a line code is used in commercial GEN2 RFID\([7] \), selecting between FM0 and Miller. Under FM0 coding, observing 2T signal duration for each bit (of duration \( T \)) suffices for BER-optimal, coherent (differential) detection and \( s_n(\cdot) \) is a \( T/2 \)-shifted waveform found in \([7] \ \text{Eq. (3)} \).

After DC-blocking, assuming perfect synchronization, the optimal demodulator projects the received signal onto the basis functions subspace using two correlators. The discrete baseband signal, at the output of the correlators, follows \([7] \ \text{Theorem 1} \):

\[
y_n = g s_n + w_n, \quad n = 1, 2, \ldots, N,
\]

where \( g \triangleq L \sqrt{\rho_n P_{R}} h^2 (\Gamma_0 - \Gamma_1) \), and \( s_n \) is the vector representation for the \( n \)-th transmitted signal. For RFID systems, which employ \( T/2 \)-shifted FM0 line-coding, \( s_n \in \{[1 \ 0]^T, [0 \ 1]^T\} \) and \( w_n \sim CN(0, \sigma^2 I_2) \) \([7] \ \text{[2]} \), with \( \sigma^2 \) denoting the variance of each noise component.

### IV. Reader

#### A. Bit Error Rate (BER)

Assuming coherent ML differential detection (with signal of 2T duration, given known channel \( g \)), the conditional bit error probability for the baseband signal in Eq. (10) follows from \([7] \ \text{[2]} \):

\[
\mathbb{P} \text{(error | } g) = 2 Q \left( \frac{|g|}{\sigma} \right) \left( 1 - Q \left( \frac{|g|}{\sigma} \right) \right),
\]

where \( Q \) is the tail probability of the standard normal distribution.
where \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-t^2/2} dt \) is the Q-function. Interestingly, a similar expression applies to Miller line coding, when the receiver performs coherent (ML) bit-by-bit detection.

### B. Outage Scenarios

The reader receives successfully the RFID tag’s information when: a) the input RF power at the tag antenna is above RF harvesting sensitivity, and b) the harvested power is above tag’s power consumption, given that the RFID tag does not include energy storage elements, and c) BER at the reader is below a predefined precision \( \beta \). Setting \( R(x) \triangleq 2Q(x)(1 - Q(x)) \), \( x \in (0, \infty) \), this event can be mathematically expressed as \([8]\):

\[
\mathbb{P}(\mathcal{C}) \triangleq \mathbb{P}(P_{in} \leq \sqrt{P_R}(\sigma R^{-1}(\beta)) | \Gamma_0 - \Gamma_1 | | \sqrt{P_u} ) = \mathbb{F}_{P_{in}}\left( \sqrt{P_R}(\sigma R^{-1}(\beta)) | \Gamma_0 - \Gamma_1 | | \sqrt{P_u} \right),
\]

where \( R^{-1}(x) = Q^{-1}\left(\frac{1 - (1 - 2x)^{1/2}}{2}\right) \), defined for \( x \in (0, 0.5) \) and \( Q^{-1}(\cdot) \) is the inverse of Q-function.

### C. Probability Of Successful Reception

Tag information is unsuccessfully received when either of previously discussed events \( \mathcal{A}, \mathcal{B}, \mathcal{C} \) occurs. Assuming that function \( p(\cdot) \) is strictly increasing and continuous around \( P_c \) and denoting for an event \( \mathcal{D} \) its complement as \( \mathcal{D}^c \), the probability of unsuccessful SWIPT reception, denoted as event \( \mathcal{F} \), can be expressed as:

\[
\mathbb{P}(\mathcal{F}) = 1 - \mathbb{P}(\mathcal{F}^c) = 1 - \mathbb{P}(\mathcal{A}^c \cap \mathcal{B}^c \cap \mathcal{C}^c) = 1 - \mathbb{P}(P_{in} > \theta_{\mathcal{F}}) = \mathbb{F}_{P_{in}}(\theta_{\mathcal{F}}),
\]

where \( \theta_{\mathcal{F}} \triangleq \max_{M} \left\{ P_{\text{sen}}, \frac{p(\zeta_{\text{har}})}{\zeta_{\text{har}}}, \Gamma_0 - \Gamma_1 | \sqrt{P_u} \right\} \). Consequently, successful SWIPT reception at the reader, under Nakagami fading, is given in closed form as follows:

\[
\mathbb{P}(\text{SWIPT success}) \equiv \mathbb{P}(\mathcal{F}^c) = \frac{\Gamma(\frac{M}{\Gamma_0 - \Gamma_1 | \sqrt{P_u}} \theta_{\mathcal{F}})}{\Gamma(M)}.
\]

### V. Numerical Results

For the simulation results the path-loss model of Eq. (1) is considered with \( \nu = 2.3 \) and \( \lambda = 0.3456 \) (UHF carrier frequency), and tag antenna reflection coefficients \( \Gamma_0 \) and \( \Gamma_1 \) satisfying \( |\Gamma_0 - \Gamma_1| = 1 \). The ultra-sensitive harvester in [2] is tested using parameters \( \tau_{\text{gel}} = 0.5 \), \( \chi = 0.5 \), \( \rho_u = 0.01 \) for RF harvesting and backscattering at the tag, while BER threshold...
is set $\beta = 10^{-5}$; variance of noise at the reader was set to $10^{-11}$.

Fig. 4 depicts probability of successful SWIPT reception at the reader, as a function of tag’s power consumption, in a strong LoS scenario (Nakagami parameter $\kappa = 10$), $d = 4$ m, and $P_{\text{R}} = 1$ Watt. Fig. 5 examines the same relationship in a non-LoS scenario ($\kappa = 2$), $d = 7$ m, and $P_{\text{R}} = 2.5$ Watt.

Both figures clearly show that the performance of the piecewise linear model $p_1(\cdot)$ coincides with the exact (ground-truth, $p(\cdot)$), data-driven model. The performance of $p_1(\cdot)$ (L), as well as $p_2(\cdot)$ (CL) model deviate from reality, even though the best values for the efficiency parameters were utilized (i.e., values that offered performance as close as possible to the ground-truth model). Both nonlinear sigmoid models tend to overestimate the event while the one incorporating sensitivity, offers closer-to-reality results in the LoS scenario and deviates further in the non-LoS scenario. Finally, the second-order polynomial $p_3(\cdot)$ underestimates performance, with performance gap that depends on the scenario and tag’s power consumption, whereas energy harvesting model $p_0(\cdot)$ overestimates the harvested power. In short, SWIPT research requires accurate energy harvesting models, otherwise misleading conclusions are unavoidable.

VI. CONCLUSION

SWIPT research should always take into account all the non-ideal characteristics of the RF energy harvesting system; otherwise, oversimplification due to overlooking fundamentals from electronics and microwave engineering may lead to impractical results. This work studied the sensitivity and the nonlinearity of the harvester. Impact of other modules, present in the RF harvesting chain (e.g., boost converter/maximum power point tracking-MPPT), should be also examined.

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